A STUDY OF PHASE MEASURING CIRCUITS AND TECHNIQUES

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1953

M-106

A STUDY OF

PHASE MEASURING CIRCUITS AND TECHNIQUES

By

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Submitted in partial fulfillment
of the requirements
for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

United States Naval Postgraduate School
Monterey, California
1953

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ENGINEERING ELECTRONICS

for the
United States Postgraduate School

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TABLE OF SYMBOLS AND ABBREVIATIONS

a,b,c,k	- Coordinates, points of intersection, constants
A,B,C,	- Coordinates, points of intersection, amplitudes, constants
ω	- Electrical angular frequency
ϕ	- Electrical phase angle
ψ	- Angular difference between electrical phase angles
ϵ	- An incremental error
s,g,h,p,	- Design parameters
K	- Gain or Amplitude factor
θ	- Absolute mechanical or electrical angle
τ	- Time increment
T	- Period, time constant
V, E	- Peak voltage amplitude
v, e	- Instantaneous voltage as function of position, time or both
Z	- General expression foran impedance
μ	- Incremental amplification factor, design parameter
ℓ	- Length, incremental inductance
f_o	- Geometrical mean frequency in a frequency band
$f_{1,2, etc.}$	- Resonant frequency

CHAPTER I

INTRODUCTION

Summary

In recent years the measurement and control of phase has become of increasing importance to communications engineering. This is particularly true in the military fields where equipments such as monopulse radar, guided missles, radio navigational systems, and sonar gears for both submarines and surface ships have become highly developed. Many important non-military applications also exist, among which are the outphasing system of single sideband radio telephony and certain systems of commercial aircraft radio navigation.

It has been observed that most texts on radio measurements and radio engineering treat the subject of phase measurement at the best in a prefunctionary manner. There is, however, a great deal of information available in the literature. This paper proposes to present a consolidated discourse on available information on the measurement and control of phase. The basic intent is to provide the student or engineer with enough of the theoretical and practical information on the subject to resolve most problems encountered.

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It was originally intended to extend the topic of phase measurement to include the microwave region. When it becomes apparent that the topic was going to be far too broad for a paper of this scope it was restricted to cover only circuits with lumped constants. Strict classification is to be expeced also in connection with certain equipments

operating in the microwave region by the nature of their operational uses. Unfortunately this often acts to limit discussion on basic principles which in themselves are unclassified; this was another reason for excluding microwave circuits.

A great deal of first hand information on the practical aspects of phase handling circuits was obtained during a ten week industrial tour with the Pacific Division of the Bendix Aviation Corporation at Los Angeles, California. During this time numerous occasions arose in which a simple four terminal device which would accurately and directly measure the phase relationships between two voltages would have been welcomed.

There are three general approaches to the problem of phase measurement: 1) the visual display of the signal and reference waveforms thereby obtaining the phase relationship by means of physical measurements applied to these waveforms, 2) the conversion of phase difference into an analog of current or voltage, and 3) the insertion of a calibrated phase shift in series with the signal to bring it into phase coincidence with the reference as noted by some phase null indicator.

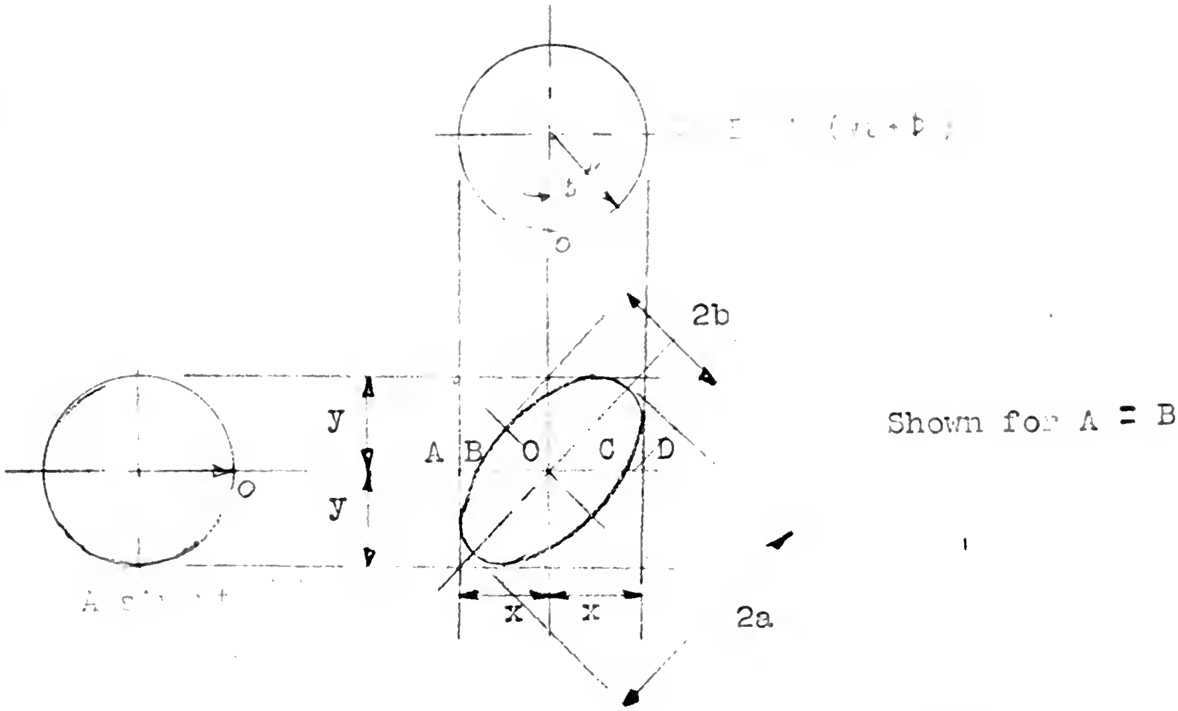
The visual methods known are restricted to single frequency sinusoidal signals. The concepts involved and equipment required are relatively simple and the accuracy obtained is not good except in certain special cases.

Electronic phasemeters are not in general use. In fact, the few reasonably accurate phasemeters known are complex and bulky enough to be called stationary. All electronic phasemeters must perform one or more of the following functions:

1. Remove amplitude variations from reference and measured signal.
2. Shift the phase of either or both reference and measured signal by some fixed amount.
3. Provide for null indication at the condition of phase coincidence or be equipped with a discriminator circuit which responds to magnitude and sense of phase difference.

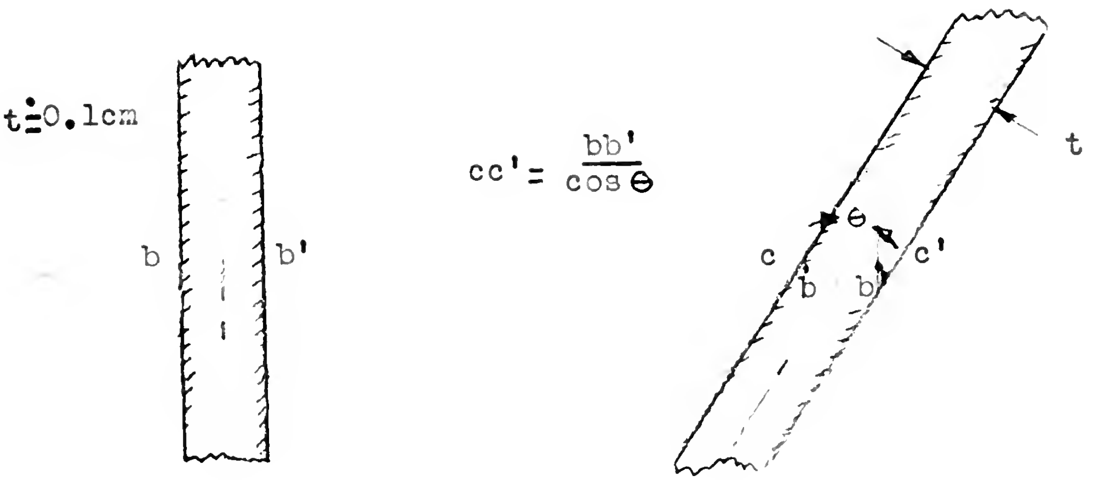
Calibrated phase shifters are obtainable and capable of very accurate single frequency performance. Like the visual methods they are of no use for complex voltages. Also the frequency division required by some of these shifters is troublesome. Their most useful applications are for use as standards and in systems having a very carefully controlled single frequency signal. The most important precision phase shifter is the goniometer, widely in use in the field of radio navigation.

Going back to the considerations of the electronic phasemeter, in the last few years a tremendous amount of work has been done in fields of phase shift networks and phase discriminators. It is the attempt of this paper to present the most important data on these subjects. Amplitude limiting circuits have been in use for many years with few modifications since their applications are not numerous. Recently a very simple limiter circuit, the cathode coupled clipper, has been proposed which promises to be of importance in speed transmission systems and any other application where limiting is required up to one megacycle. Because of its simplicity and apparent superiority to any other circuit for phasemeter limiting operations it will be considered in some detail.



Lissajous Phase Pattern

Figure 1



Observation Error due to Trace Thickness

Figure 4

CHAPTER II

VISUAL METHODS OF PHASE MEASUREMENT

The visual method of presentation and comparison of electrical signals for phase measurement is perhaps the most universally used of any method possible. Contrary to first impression it is not inherently inaccurate and in certain cases it may prove to be the most accurate method possible, either practically or theoretically.

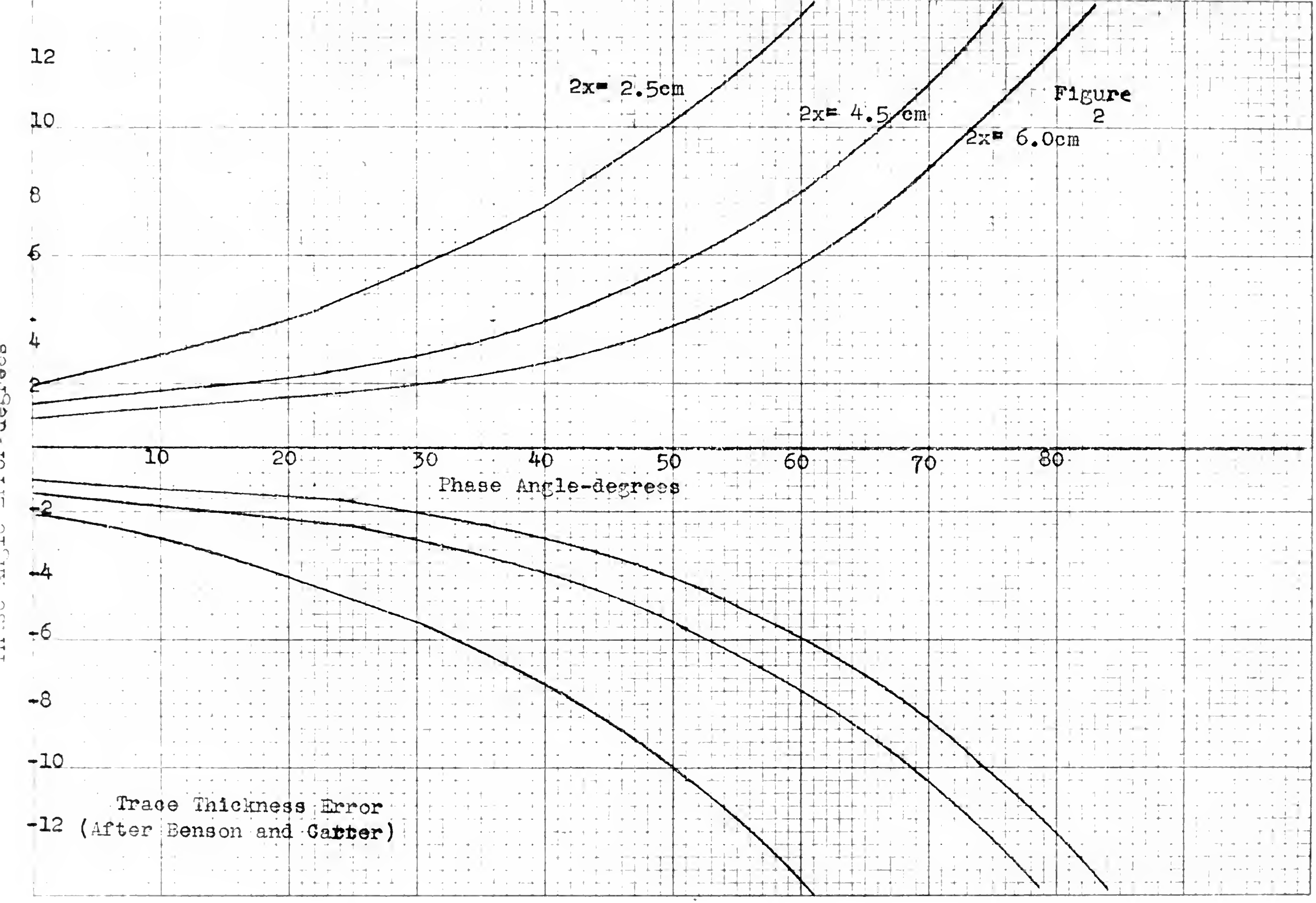
The familiar Lissajoux figure is illustrated in Figure 1. The following relationships are obtained from the figure

$$BOC = 2B\sin(\omega t + \phi)\big|_{\omega t=0} = 2B\sin\phi$$

$$AOD = 2B \qquad \frac{BOC}{AOD} = \sin\phi \tag{1-1}$$

Obviously the method is most accurate at $\phi = n\pi$ where the ellipse degenerates into a straight line. In this case extremely accurate comparison is possible (Appendix I). This method is used again in the discussion of phase standards in a later chapter.

There are two principal sources of error when the phase angle is not a multiple of π and the sine is obtained by intercept ratio. There are 1) observer error due to the CRO trace having appreciable thickness and 2) error due to deviation from an elliptical pattern with the presence of harmonics in the signal. Extensive mathematical treatment has been given to these considerations[1] and the theoretical results are shown in Figures 2 and 3. The mathematical treatment is quite long and, because of the nature of some of the assumptions used,


12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
10
20
30
40
50
60
70
80
Phase Angle-degrees
2x= 2.5cm
2x= 4.5 cm
2x= 6.0cm
Figure
2
Trace Thickness Error
(After Benson and Carter)

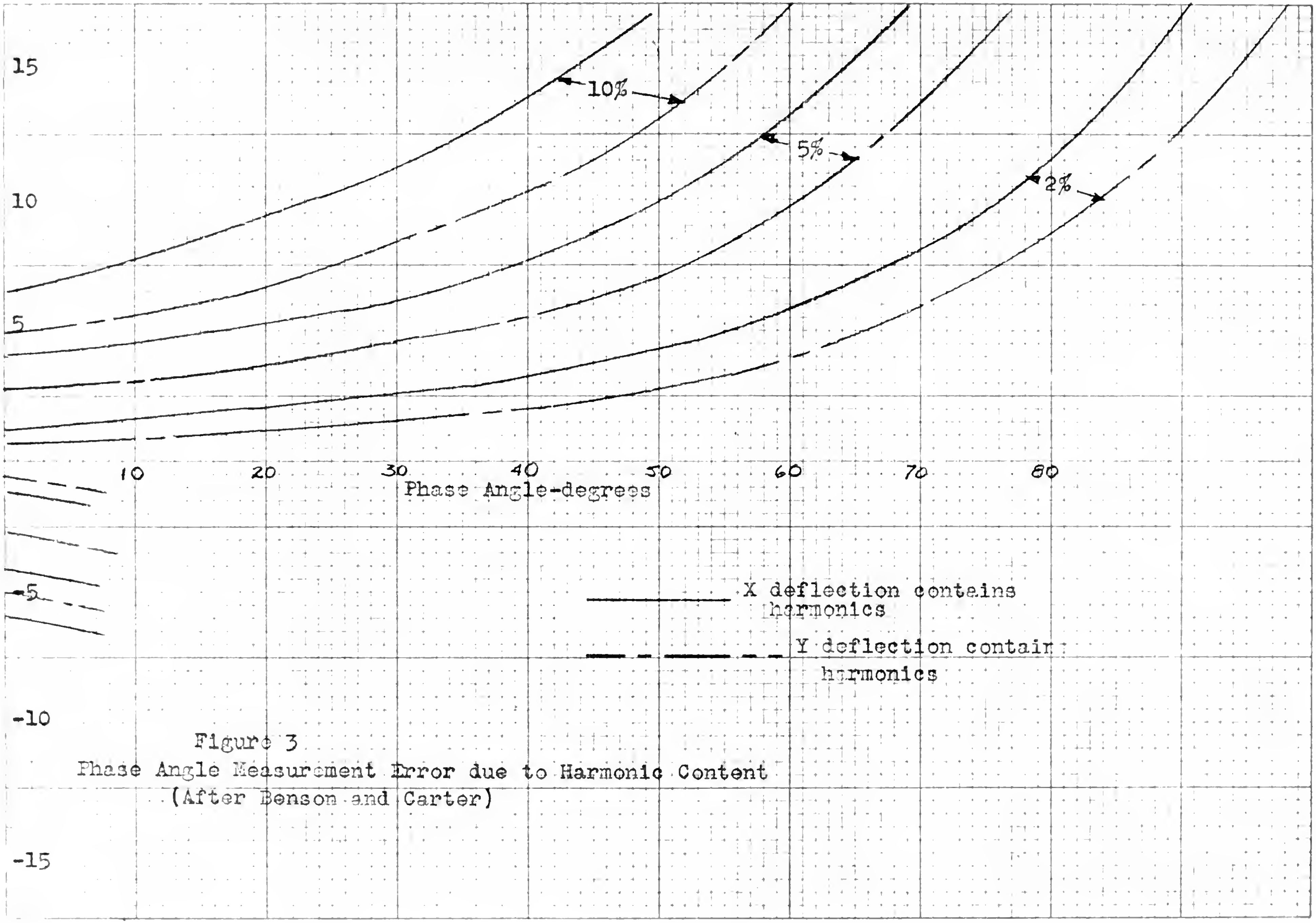


Figure 3
Phase Angle Measurement Error due to Harmonic Content
(After Benson and Carter)

was adjudged to be of too little value to be included as an appendix. The assumptions made are that the trace is a constant thickness of 1mm. and that the observer is consistently able to read the intercept within an accuracy of plus or minus 0.5mm. Then, as the angle of transit of the trace across the reference axis changes with phase angle, the effective intercept as measured also changes as is illustrated in Figure 4. This results in the theoretical error becoming a transcendental function of phase angle. Experimental results show the agreement between theory and actual measurement to be no greater than 50% at phase angles between 30 and 60 degrees.

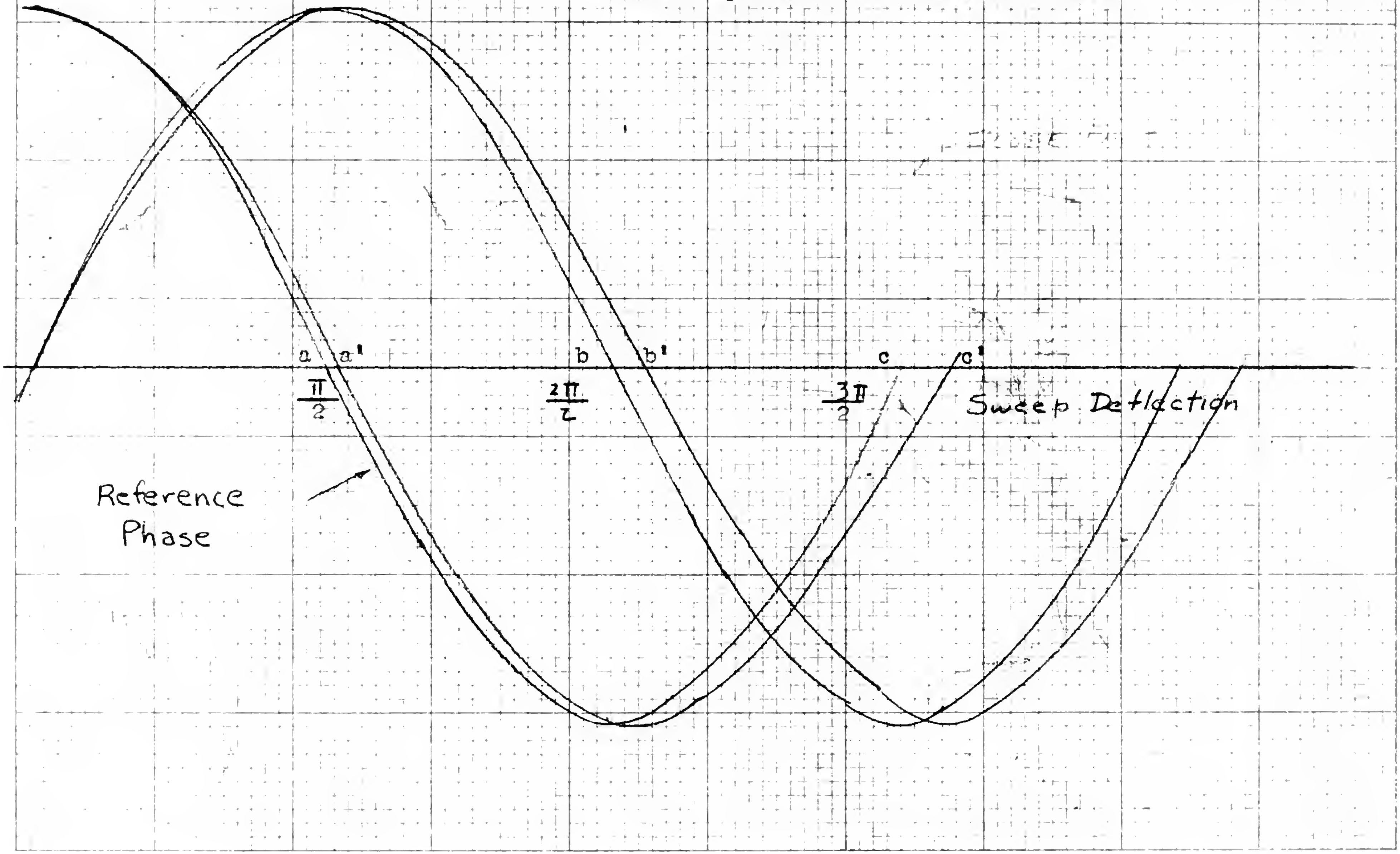
In summary of this method it may be said that with great care in making observations with a 6 inch effective diameter CRO and applying corrections, phase angles up to 45-60 degrees can be measured with an accuracy of plus or minus 1 degree. In the special case where $\phi = n\pi$ the error can become negligible and phase angles as small as 0.3 degree can be determined (Appendix I).

A second visual method which is frequently used is simultaneous visual observation of two signals by means of a dual beam CRO or single gun CRO and electronic switch. The first method is to be preferred since the several electronic switches examined have a lower limit of switching frequency which limits the minimum frequency of the signal to be examined to about 100 cycles.

Possible sources of error in measuring phase with a dual beam CRO are 1) wave distortion due to non-linear sweep, 2) amplitude distortion in the vertical amplifier, 3) observer error in locating identical phase points.

Figure 5

Effect of Non-linear Sweep on the Intercept Method of Phase Measurement



The second of these can be easily avoided by chosing the reference points as the intersection of the trace with the zero deflection axis as established by grounding the input terminals. Clearly the steeper the angle of intercept of the trace with this reference axis the smaller the observer error. This had been previously brought out in the discussion of the error due to trace thickness in the Lissajoux pattern method. Therefore, by using maximum deflection and carefully locating the axis of zero deflection two of the error sources can be minimized. There remains only the sweep nonlinearity to be considered.

Assume the two signals to be compared are sinusoidal and are displayed on a dual beam CRO so adjusted to place both traces on a common axis. For a given signal frequency and sweep length the sweep nonlinearity can be established as a function of the electrical angle θ, $f(\theta)$ say, Figure 5. Further assume that the value of the phase angle is established by the relationship

$$\phi' = \frac{ab'}{ac'} \times \pi \tag{1-2}$$

with the sweep so controlled to display one half cycle of the reference phase across the scope face. This is the maximum degree of expansion possible.

If the sweep were exactly linear the true phase angle would be given in terms of the undistorted intercepts

$$\phi = \frac{ab}{ac} \times \pi$$

since

$$ab' = ab + f(\theta_1) = ab + f(\phi)$$
$$ac' = ac + f(\theta_2) = ab + f(\pi)$$

and the error in measurement of the phase angle is

$$\epsilon = \phi' - \phi$$

in terms of the constants of the system

$$\epsilon = \pi \left\{ \frac{ab + f(\phi)}{ac + f(\pi)} - \frac{ab}{ac} \right\}$$

$$= \pi \left\{ \frac{f(\phi) - \frac{ab}{ac} f(\pi)}{ac + f(\pi)} \right\}$$

$$= \frac{\pi f(\phi) - \phi f(\pi)}{ac + f(\pi)} \tag{1-3}$$

To determine at what value of ϕ the error will be a maximum or minimum differentiate the function with respect to

$$\frac{d\epsilon}{d\phi} = \frac{\pi \frac{d f(\phi)}{d\phi} - f(\pi)}{ac + f(\pi)} \tag{1-4}$$

The simplest sweep deflection circuit is a sawtooth generator using the first portion of an exponentially varying voltage where the exponential is almost linear in form. In this case the equation for trace deflection across the scope becomes

$$x = B(1 - e^{-t/T}) \tag{1-5}$$

where B is some maximum deflection and T is the time constant of the system. Since, for any given electrical frequency and signal presentation, the physical deflection can be related to the electrical angle by

$$t = \theta/\omega$$

and

$$f(\theta) = B(1 - e^{-\theta/T\omega}) - A\theta$$

$$\epsilon = \frac{\pi[B(1 - e^{-\phi/T\omega}) - A\phi] - \phi[B(1 - e^{-\pi/T\omega}) - A\pi]}{ac + B(1 - e^{-\pi/T\omega}) - A\pi}$$

Now

$$\frac{d\epsilon}{d\phi} = \frac{\frac{B\pi}{T\omega}(e^{-\phi/T\omega}) - B(1 - e^{-\pi/T\omega})}{ac + B(1 - e^{-\pi/T\omega}) - A\pi}$$

This gives the value of ϕ at which the error in measurement is a maximum to be

$$\phi = T\omega \ln\left[\frac{\pi}{T\omega(1-e^{-\pi/T\omega})}\right] \qquad (1\text{-}6)$$

This function is indeterminate in the limits, but investigation in the region Tw = 1 to Tw = 100 indicates ϕ of maximum error to be π radians when Tw = 30. The interpretation of this is that for Tw = 30 the error is approximately proportional to the value of ϕ and for Tw greater than 30 the sweep approaches linearity and the error goes to zero. For values of Tw less than 30, ϕ of maximum error approaches zero.

A minimum condition arises when $f(\theta) = K\theta$ which can be verified by substitution into the error equation 1-3

$$e = \frac{\pi k\phi - \phi k\pi}{ac + k\pi} = 0$$

This is a trivial case since

$$ab' = ab + k\phi \qquad \& \qquad ac' = ac + k\pi$$

represent another linear sweep of a different rate.

The implications are that this method is inherently more accurate for larger values of phase angle, at least with this particular type of sweep distortion which seems the most likely.

These same expressions can be used to estimate the probable measurement error due to observational error by setting

$$ab' = ab \pm k \quad ; \quad ac' = ac \pm k$$

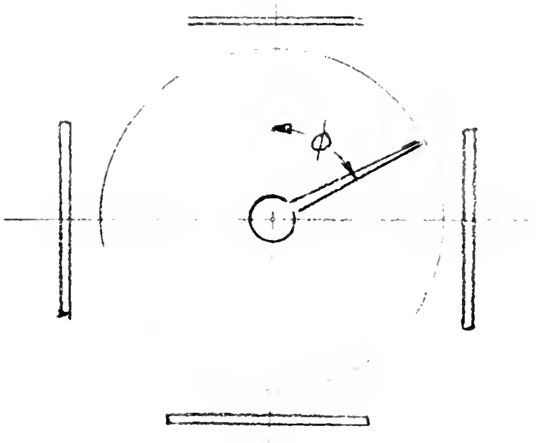
giving

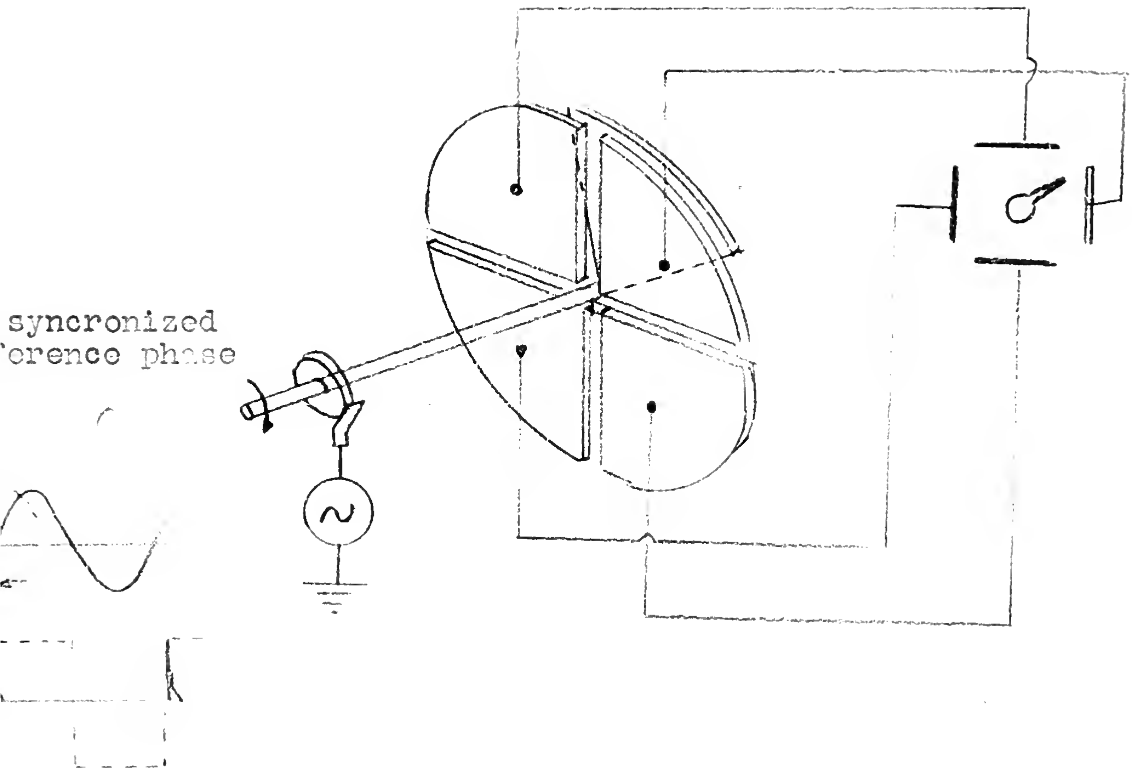
$$\epsilon = \pm\left\{\phi\left(\frac{k}{ac+k}\right) - \left(\frac{k\pi}{ac+k}\right)\right\}$$

This represents the most prevalent type of human error where the observer's measurements are consistently too large or too small. Since very linear sweeps are possible in well made oscilloscopes, it is concluded that the

Visual Phase Measurement by
Circular Trace Presentation

Figure 6





theoretical accuracy obtainable by the use of this method is of the order of magnitude

$$\epsilon \leq \frac{k\pi}{ac + k}$$

If a 6 inch usable scope face and an observer error of plus or minus 0.5 mm are chosen the error becomes 1.16 degrees maximum which is good enough for most work. Having the entire range of 0-360 displayed unambiguously is also an advantage of this method. Obviously the larger the CRO the smaller the error.

A variation of this basic method can be used with a time based sweep single gun oscilloscope when provisions can be made to trigger the sweep externally. A typical method would be as follows: the reference signal is squared, differentiated and the negative peak clipped. The spike voltage thus obtained is used to trigger the sweep upon which the signal to be measured is displayed. By use of time markers or intercept measuring technique similar to that previously discribed, the phase difference may be determined. The obvious disadvantage of this method is that the trigger signal does not pass through an identical channel as the signal to be measured and is not of the same form; phase shift inherent in the CRO amplifiers may manifest itself as an apparent phase difference between the two signals.

A third general approach to the display by CRO is by obtaining a circular trace which is then either Z-modulated or momentarily expanded to produce "spikes" as shown in Figure 6. The circular sweep may be obtained by splitting one of the signals in quadrature by an R-C network, or it

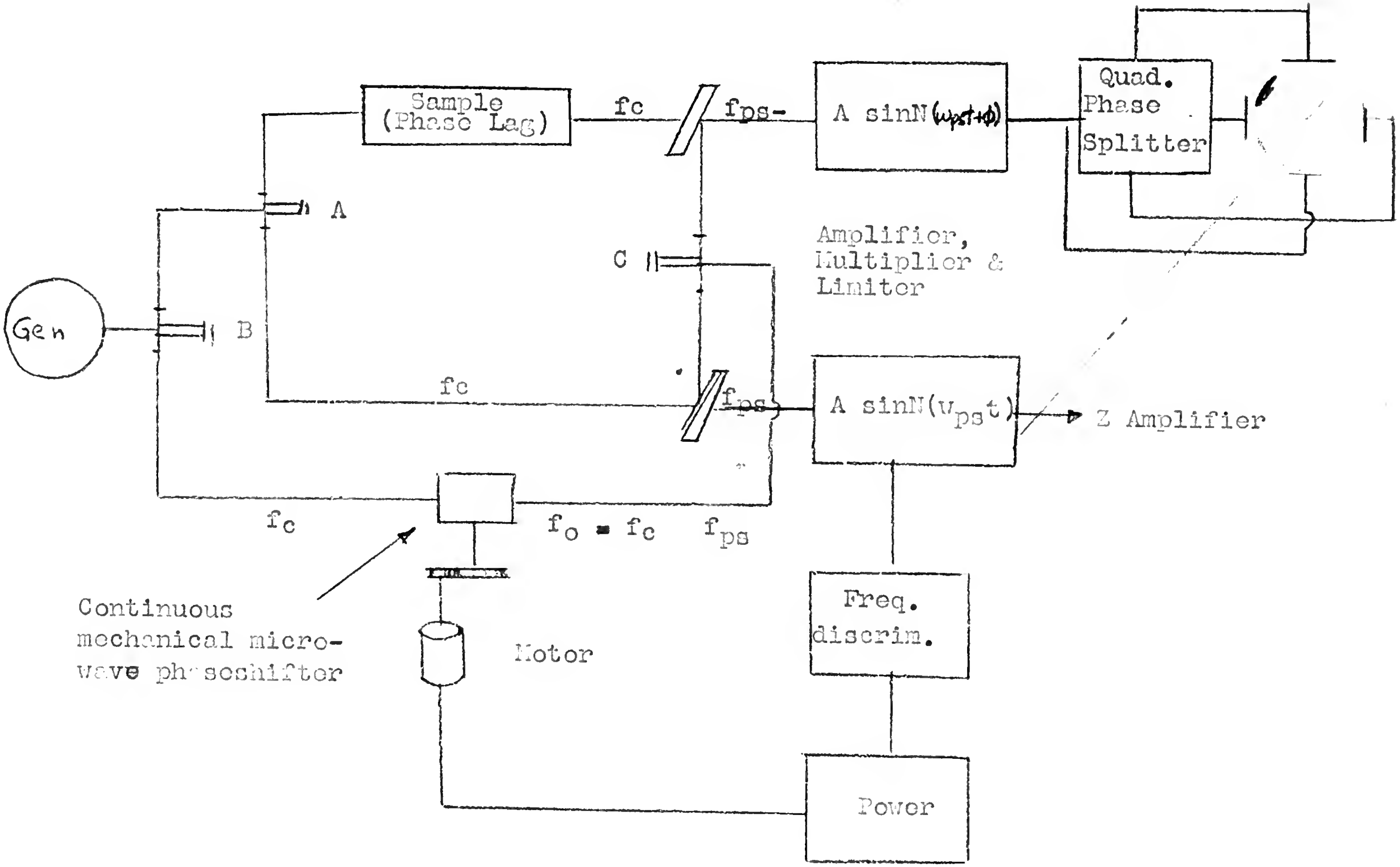


Figure 7

A Laboratory Application of Z-Modulation
Phase Measurement

can be obtained externally. These methods may be found useful when there is the possibility of multiplying the frequency by a large factor thereby multiplying the phase difference by a like factor.

An experimental device[20] developed to measure the index of refraction of air by measurement of the phase shift of an electro-magnectic wave passed through the medium is shown in Figure 7. This device uses Z-modulation of a CRO to indicate the shift. By frequency multiplication of 1875:1 very small shifts can be measured. This large shift is also necessitated by the fact that the signal is applied directly to the Z amplifier as a sine wave and the modulated beam appears as an arc rather than a spot.

Referring to the figure the signal source is a klystron operating in the region of 400 mcs. Magic Tees A, B, and C distribute the r.f. energy through the three paths shown. The signal passing through the sample of air under examination is retarded in some amount. The continuously rotating mechanical phaseshifter is driven by a source synchronized to klystron. This continuous phase shift changes the frequency by slightly more than 100 cps. This shifted signal is used as a local oscillator to hetrodyne down both the reference signal and the signal to be examined for phase shift to a useable audio frequency. The two audio frequencies are multiplied and limited, then applied to the phase measuring circuit in this case a CRO. Crystal mixers are used in the hetrodyning process.

This system is cited as a practical application of phase measuring technique, also because it illustrates the fact that phase difference can

be multiplied (or divided) by multiplying frequency, and that phase relationships remain invariant during heterodyning. The latter is not quite so obvious as the former. In the interest of complete treatment of the subject proof of this is shown in Appendix II.

The implications of these two properties are thought to hold considerable importance to the problem of phase measurement. At the expense of suitable circuitry for frequency multiplication or division any degree of accuracy desired can be obtained. Also, when the space considerations predominate, a local oscillator and lumped constant discriminator may provide space savings over a microwave discriminator. In any event, it puts the upper limit of frequency for any satisfactory phase meter at that frequency which can be heterodyned into the useful range of the instrument.

R-C Lattice Phaseshift Network
(Dome Network)

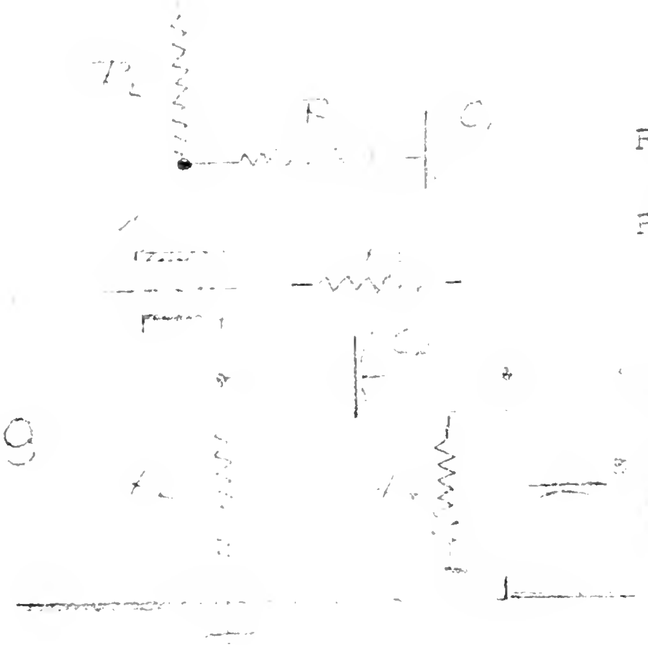


Figure 8

$R_1C_1 = R_2C_2 = R_3C_3 \qquad C_2 = aC_1 \qquad R_2 = \frac{R_1}{a}$

$R_3 = \left[\frac{1-4a}{4a}\right] R_2 \qquad C_3 = \left[\frac{4a^2}{1-4a}\right] C_1$

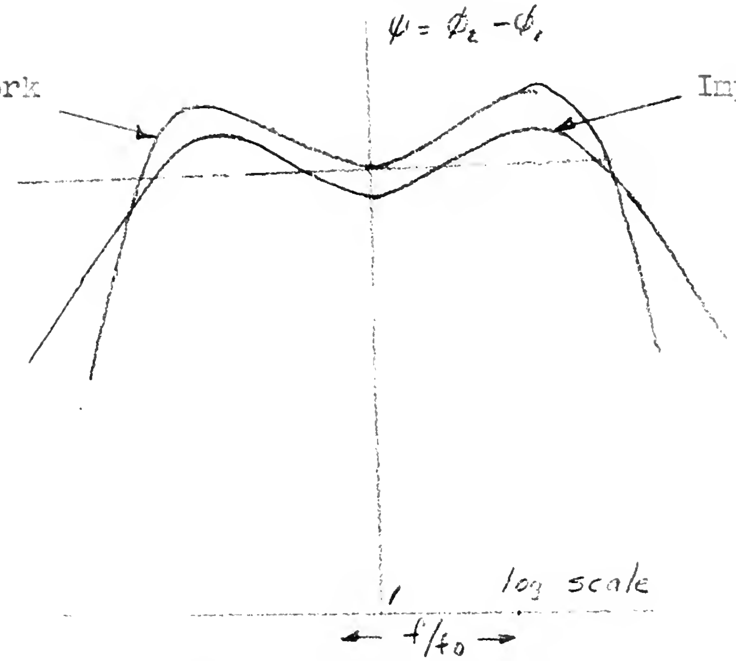
$f_0 = \frac{1}{2\pi R_1 C_1} \qquad a = \frac{1}{s+2}$

$s \geq 2 \qquad s = \frac{1-2a}{a}$

$$|e_0| = (1-4a)|e| = \frac{(s-2)}{(s+2)}|e|$$

$$\phi = \tan^{-1} \frac{2sff_0(f^2 - f_0^2)}{(f^2 - f_0^2)^2 - sf^2 f_0^2}$$

Figure 9



Output Phase Characteristic for a Pair of Networks Used to Provide a Constant Phase Difference over a Wide Band of Frequencies

CHAPTER III

CONSIDERATIONS OF AN ELECTRONIC PHASEMETER

As previously mentioned an electronic phasemeter may perform any or all of these three functions 1) phase shift of a predetermined amount in either or both channels, 2) removal of amplitude variations in the signals, 3) discrimination of the phase to produce an analog output of current or voltage.

1. Phase Shifting

When the phase shift desired is 180 degrees it may be obtained at once by a vacuum tube inverter or by a transformer. When the shift desired is something other than 180 degrees complications arise. It is not hard to devise a circuit to create a continuously varying output phase, but these circuits generally have the unpleasant quality of a continuously varying output amplitude. The consideration of a passive, constant attenuation, broadband phase network has received considerable attention in recent years particularly in view of its applications to single sideband radio transmission. Because of its inherent importance to the problem of broadband phase measurement, it is not considered too much of a digression to treat these networks in some detail here.

The first intensive treatment of such a network was made by Dome[4]. He proposed the half lattice R-C network illustrated in Figure 8, and developed the necessary relationships between circuit parameters. Described briefly the circuit exhibits a phase characteristic which is proportional to the logarithm of the signal frequency. When two of these

networks are used in conjunction the difference in the phase angles of their output voltages may be held constant over a considerable range of frequency.

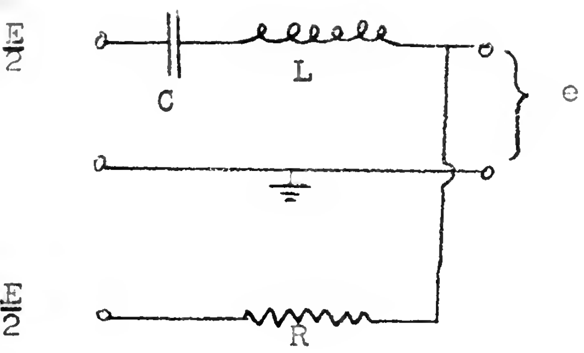
$$\begin{aligned}
\phi_1 &= \ln f + C \\
\phi_2 &= \ln kf + C \\
\phi_2 - \phi_1 &= \ln kf - \ln f \\
&= \ln k \\
&= \text{Constant}
\end{aligned} \tag{2-1}$$

When using a pair of these networks to obtain a constant phase difference Dome proposed that at f_o, the geometrical mean frequency of the required operating frequency band, the respective phase shifts of the two networks be 180 degrees plus or minus 45 degrees, assuming the phase constant to be 90 degrees for the purposes of illustration. A careful plot of the phase difference between the networks shows the maximum deviation from the required difference to occur near the upper and lower limits of the frequency band, Figure 9. Obviously if the maximum phase excursion is not qualified by a sense restriction there is no reason to restrict the phase difference error to zero at the mean frequency. Laden in an unpublished thesis at the U. S. Naval Postgraduate School[11] attacked the problem from the standpoint of the theory of a function of a real variable. By manipulation of the circuit equations he obtained the equation

$$\phi_\beta - \phi_\alpha - 90° = 2\left(\tan^{-1}\frac{S_\beta \omega \omega_{o\beta}}{\omega^2 - \omega_{o\beta}^2} - \tan^{-1}\frac{S_\alpha \omega \omega_{o\alpha}}{\omega^2 - \omega_{o\alpha}^2} - 45°\right) \tag{2-2}$$

which bears a marked similarity to Dome's equation. By treating this function to produce a minimum maximum the following relationships between

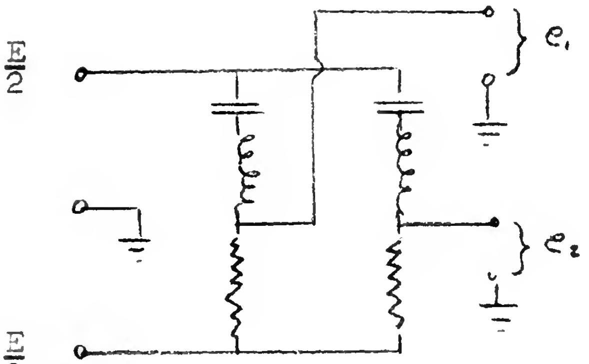
A Basic L-C-R Constant Attenuation Phase Shift Network



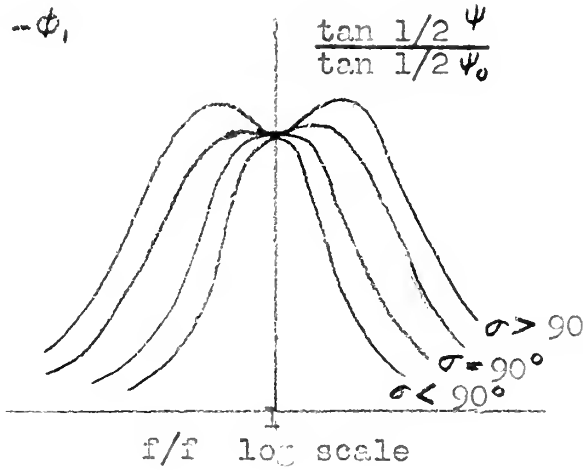
$w_1^2 = 1/LC$
$Q^2 = L/CR$
$k = 1/2$
$L = QR/w_1$
$C = 1/w_1 QR$

Figure 11

Characteristics of a Constant Phase Difference Network (After Luck)



$\psi = \phi_2 - \phi_1$



circuit parameters are found to hold.

$$\omega_{0\alpha} = g\Omega \qquad \omega_{0\beta} = h\Omega \qquad \Omega = \sqrt{\omega(\text{upper f.})\,\omega(\text{lower f.})}$$

$$p = \sqrt{\frac{\omega(\text{upper})}{\omega(\text{lower})}} \qquad u = h - g \qquad h = 1/g$$

$$u = \sqrt{(p+1)\left[\sqrt{p^2+1} + \sqrt{2}\,p - (p+1)\right]/p}$$

These relationships enable one at once to choose the optimum circuit parameters having decided upon the band width.

An article by Luck which appeared at the same time gives a much clearer picture of the problem since he arrives at exactly the same results as Laden with none of the ponderous mathematical treatment[12]. Luck's results must be used in graphical form and do not provide an analytical expression for circuit parameters. Starting with the simple L-C-R lattice of Figure 10 Luck developed an expression for the complex ratio of output to input voltage and used this expression as a basis of all subsequent development. The output voltage of the circuit of Figure 10 is

$$e = -\frac{E}{2} + \frac{RE}{R + j\omega L + 1/j\omega C}$$

using the conventional expression for the quality of a circuit

$$Q = \frac{\omega_1 L}{R} = \frac{1}{\omega_1 RC} \qquad (\omega_1 \text{ is resonant freq.})$$

and dividing through by R and simplifying the resultant expression for voltage transfer becomes

$$\frac{e}{E} = K\,\frac{1 + jQ\left(f_1/f - f/f_1\right)}{1 - jQ\left(f_1/f - f/f_1\right)} \qquad (2\text{-}3)$$

To evaluate the phase angle between input and output voltages let

$$Q\left(f_1/f - f/f_1\right) = \tan 1/2\,\Phi$$

by substitution of this value into equation 2-3 it is seen that the phase angle between input and output is exactly $\emptyset$.

Now if two of these simple lattices are constructed with different values for $\emptyset$ the phase difference is $\psi = \emptyset_2 - \emptyset_1$. The expression for the tangent of the half angle becomes

$f_1 = f_0/r$
$f_2 = f_0 r$

$$\tan \psi/2 = \frac{Q\left[\left(\frac{r f_0}{f} - \frac{f}{r f_0}\right) - \left(\frac{f_0}{r f} - \frac{r f}{f_0}\right)\right]}{1 + Q^2\left[\frac{r f_0}{f} - \frac{f}{r f_0}\right]\left[\frac{f_0}{r f} - \frac{r f}{f_0}\right]} \tag{2-4}$$

At this point Luck ingeniously introduces four parameters

1) $f/f_0 = \tan 1/2\, n$

2) $Q\left(r - \frac{1}{r}\right) = \tan 1/4\, \psi_0$

3) $\frac{1}{Q^2} - \left(r - \frac{1}{r}\right)^2 = 4\cos\left(\sec^2 1/2\, \sigma\right)$

4) $\tan 1/2\,\sigma \operatorname{cosec} n = \tan 1/2\,\theta$

By substitution of these values and reduction of the resulting equations the very simple equation for the behavior of the phase difference network is obtained.

$$\frac{\tan 1/2\, \psi}{\tan 1/2\, \psi_0} = \frac{\sin \theta}{\sin \sigma} \tag{2-5}$$

Using this expression the family of curves shown in Figure 11 is obtained which bear resemblance to the familiar universal resonance curves for a single tuned coupled circuit. By inspection of these curves it is obvious that for a specified maximum of phase deviation from a given constant phase difference, Q and r should be chosen to place the value of ψ_0 at somewhat less than the phase constant and n somewhat more than 90 degrees in order to achieve optimum bandwidth. The parameters used by Dome in his development corresponds identically with Q in its conventional

sense as used here.

Luck further reduced a number of similar lattice structures to the basic circuit used in this development and these are illustrated in the appendix (Appendix II). The fourth of these circuits is the one originally proposed by Dome. In terms of Luck's analysis the conditions required by Dome exist when

$$\psi_0 = 90^\circ$$

This network takes precedence over the others by virtue of the fact it can tolerate a load of resistive, capacitive, or inductive nature.

From experience with practical uses of this network, the following comments are made about the basic design (see Figure 8):

1) The tolerance of the two series arms are very critical; components used in these impedors should be of plus or minus 1%.

2) The value of the resistance in the shunt arm is also critical, but for some reason the capacitance is not. This is very fortunate since it permits feeding the output signal from the network directly into an amplifier stage. If a pentode amplifier is used the input capacitance may be quite large. In one particular instance the resistance R3 was 62.5 k and C3 was made variable from 480-575 mmf.

3) The equality of voltages in the phase splitter is not as critical as might be expected. The plate and cathode load resistors can be made plus or minus 5% tolerance without undue error being noticed.

The above observations are strictly of an experimental nature and no analytical explanation is offered.

The R-C feedback amplifier can also be used to obtain a precise 90 degree phase shift. While this device can be adjusted to give exactly

Figure 12
General Feedback Amplifier

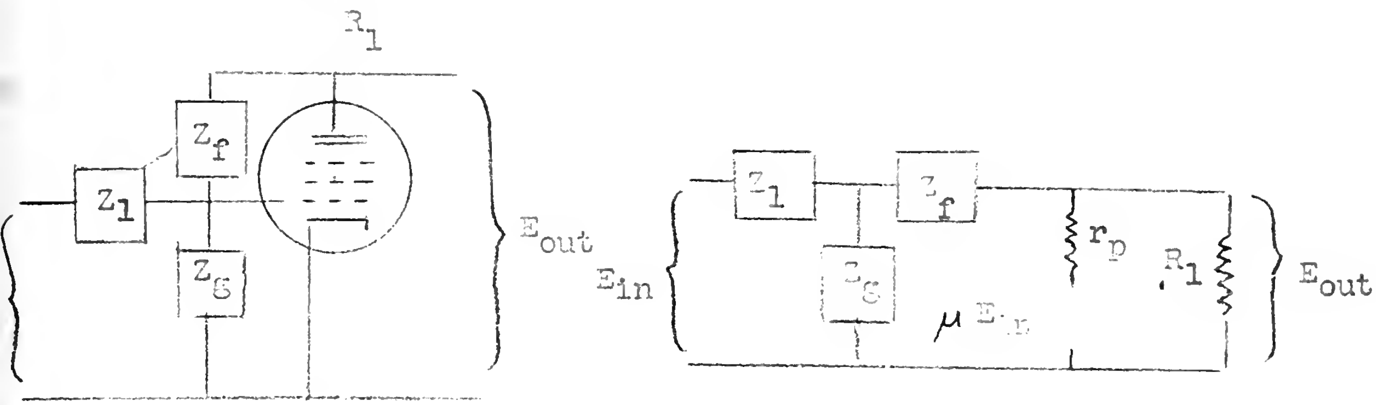
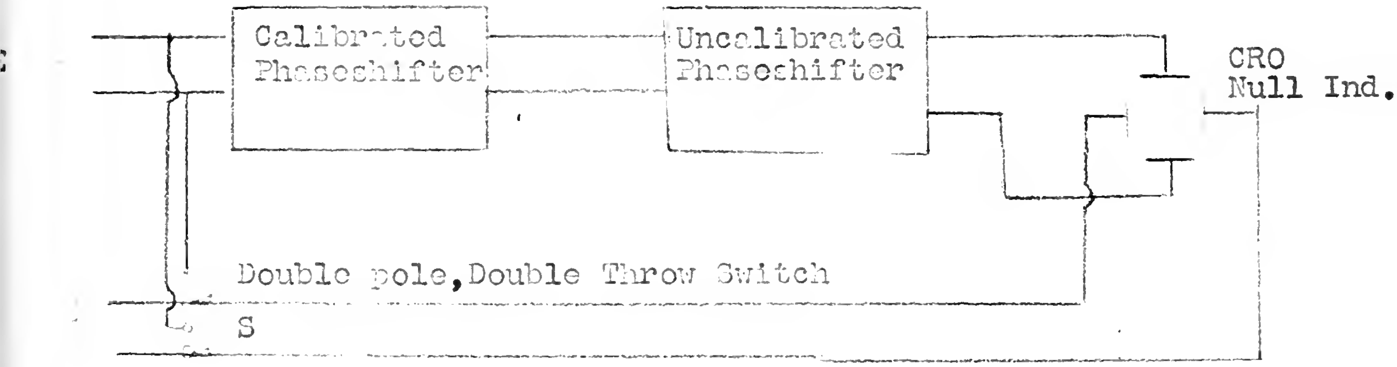
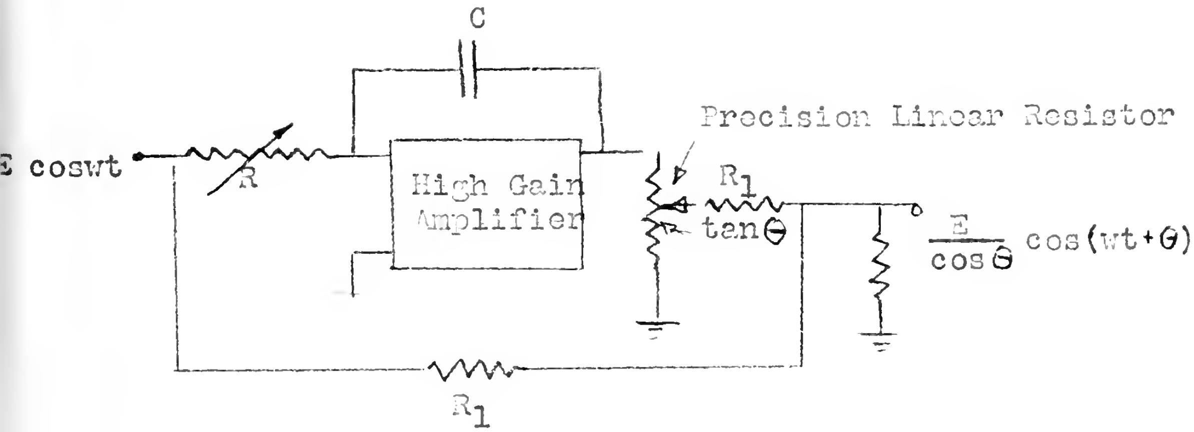


Figure 13
An R-C Feedback Amplifier Precision Phasemeter





90 degrees of shift at some particular frequency, it is very frequency sensitive and cannot be used when the signal to be shifted is complex. This system is used in a certain highly precise phasemeter which, in view of its purchasers, has been used almost exclusively in the field of radio navigation.[10]

The manner in which the shift is obtained is by adjusting the feedback network to provide exactly unity gain. With this condition the phase shift of the amplifier is 90 degrees. To verify this statement consider a single, high gain vacuum tube amplifier, Figure 12, arranged as a general feed back circuit. Following the analysis of Seely, page 155 op. cit, the complete expression for the gain of the circuit is

$$K_r = -\frac{Z_f}{Z_1} \frac{1}{1 - \frac{1}{K} - Z_f \frac{Z_1 + Z_g}{Z_1 Z_g} \times \frac{1}{K}} \qquad (2\text{-}6)$$

and if Zg can be taken as being very much greater than Z1

$$|K_r| = \frac{Z_f}{Z_1} \times \frac{1}{1 - \frac{1}{K} - \frac{Z_f}{Z_1 K}}$$

For a multistage amplifier very high voltage gains can be realized. The terms containing the factor 1/K can be assumed to vanish and the resultant expression for gain is

$$K_r' = -\frac{Z_f}{Z_1}$$

When the gain is reduced to unity Z_f equals Z_1 and the complex gain equation is

$$K_r = -\frac{1/j\omega C}{R} = -\frac{1}{j\omega CR} = \frac{1}{\omega RC} \angle 90^\circ$$

showing that a phase shift of exactly 90 degrees is obtained.

An interesting direct application of the R-C feedback amplifier in phase measurement was made by Raggazzini and Zadeh.[17] A simplified diagram of their wideband audio phase meter is shown in Figure 13. The output of the phase shift amplifier is combined with the input voltage to give a resultant voltage which is directly related in phase to the mechanical angle of rotation of the precision resistance potentiometer R.

$$E_1 \cos \omega t + [E_1 \tan \theta][\cos(\omega t + \pi/2)] = \frac{E_1}{\cos\theta} \cos(\omega t + \theta) \quad (2\text{-}7)$$

The application of this device is straightforward. With the calibrated phase shifter set at zero the uncalibrated phase shifter, which in this case is an R-C bridge, is adjusted to produce phase coincidence as indicated on the null indicating device. For null indication a CRO is used taking advantage of the expanded ellipse technique discussed in Appendix I. Switch S is then thrown to its alternate position and the calibrated phase shifter adjusted to regain the null condition. Since θ can be varied only $\pm$ 45 degrees by the precision potentiometer, additional $\pm$ 45 degrees of phase shift must be adjudged by noting the slant of the elliptical pattern.

This principle is utilized in the precision phase shifter referred to earlier with the following refinements:

1) A low level wattmeter is used to resolve ambiguity about 180 degrees giving the instrument an effective range from zero to 360 degrees phase shift.

2) A diode bridge is used as an indicating device instead of a CRO.

2. Wave Squaring and Limiting Circuits

Phase discriminators are normally amplitude sensitive, and it

Figure 14

A Pentode Wave Squaring Circuit

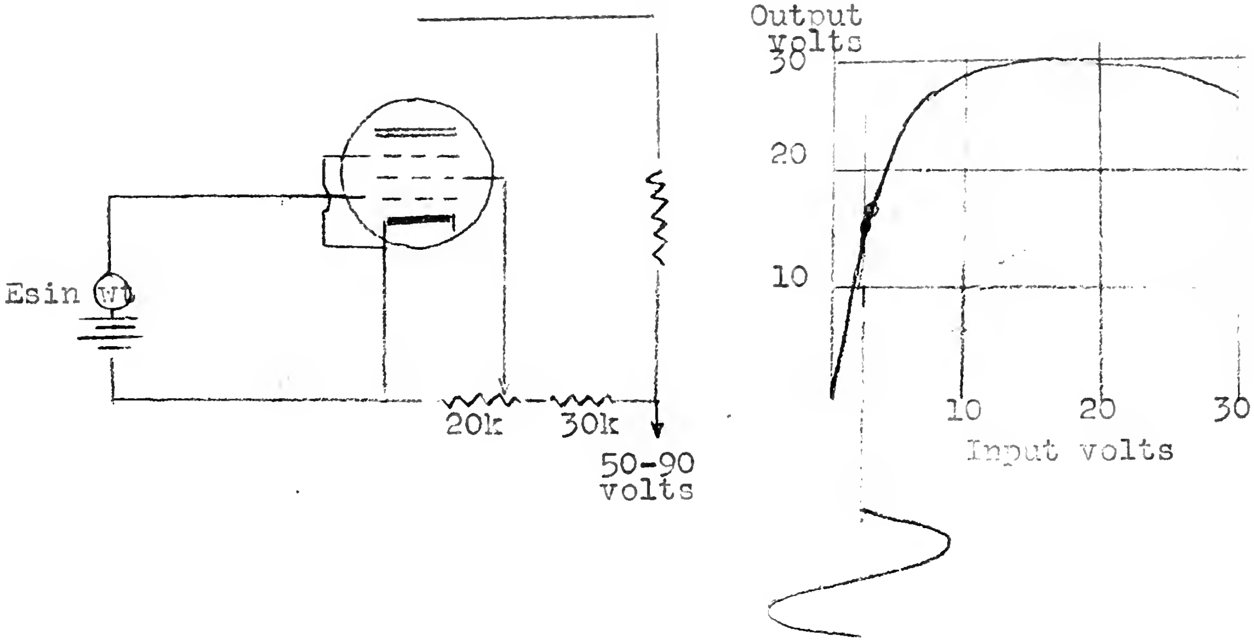
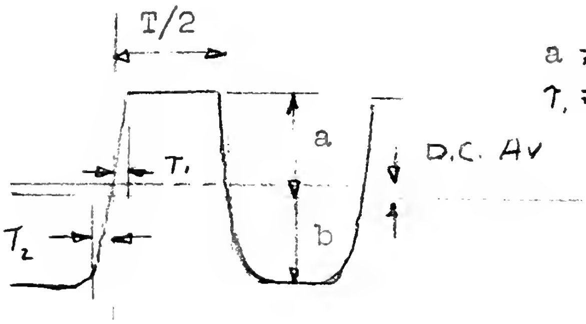
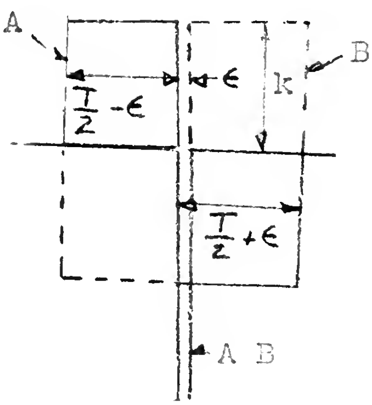


Figure 15



$a \neq b$
$T_1 \neq T_2$

Due to improper location of quiescent point



Signal after further amplification and limiting showing that upon combination with another symmetrical signal 180 degrees out of phase a voltage

$$= \frac{\epsilon \times 2k}{T}$$

indicative of phase deviation from 180 degrees shift is produced.

Indicated Phase Error due to Dissymettryin Wave Squaring

becomes necessary to remove amplitude variations in the reference and compared signal prior to discrimination. Possible circuits for signal limiting include diode clippers, multigrid tubes with either or both saturation and grid circuit limiting, pentode-diode combinations, synchronized multivibrators, etc. The most widely used limiter circuit has been a pentode amplifier operated in the region of saturation, a circuit generally associated with FM receivers, Figure 14.

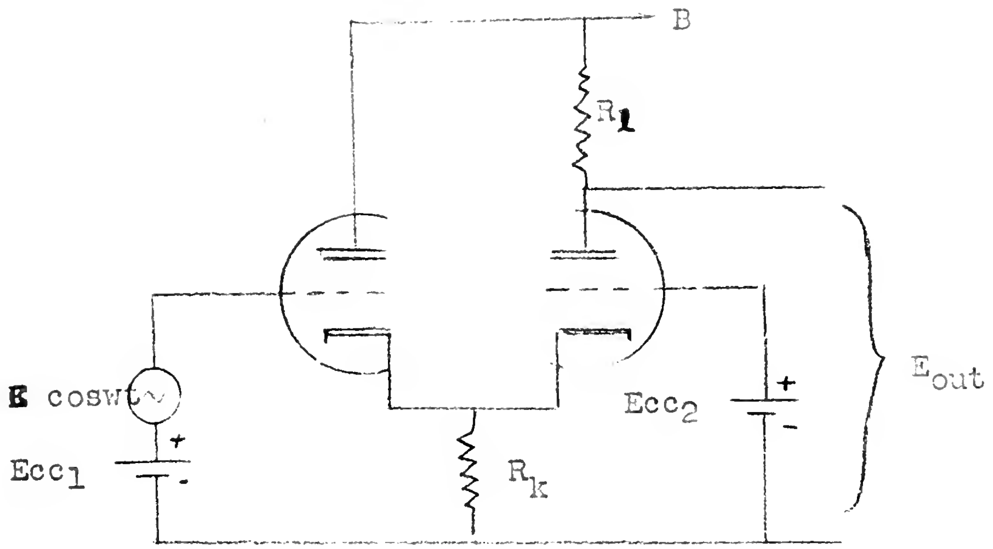
When identical limited waves must be produced in two channels operating conditions of the two limiter circuits must be very carefully controlled least dissymmetry be produced. This will result in an apparent phase difference out of the discriminator. Consider the condition illustrated in Figure 15; in this case a summing type of phase discriminator will produce a phase difference voltage directly proportional to the degree of dissymmetry.

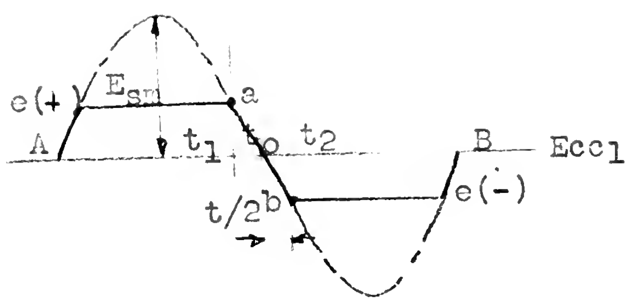
A limiting circuit satisfactory up to several hundred kilocycles which is substantially independent of circuit and tube tolerances and changes is the cathode coupled clipper[6], Figure 16. This very useful circuit should be considered anytime an audio limiting circuit is needed. Because of the importance of a satisfactory limiting circuit, not only in connection with phase measurement but as applied to speech handling equipment such as compressors and companders, this circuit is considered in detail in Appendix III.

Certain design considerations must be taken into account in the application of this circuit. When the signal levels obtainable are low

Figure 16

The Cathode Coupled Clipper Circuit





Clipping Action

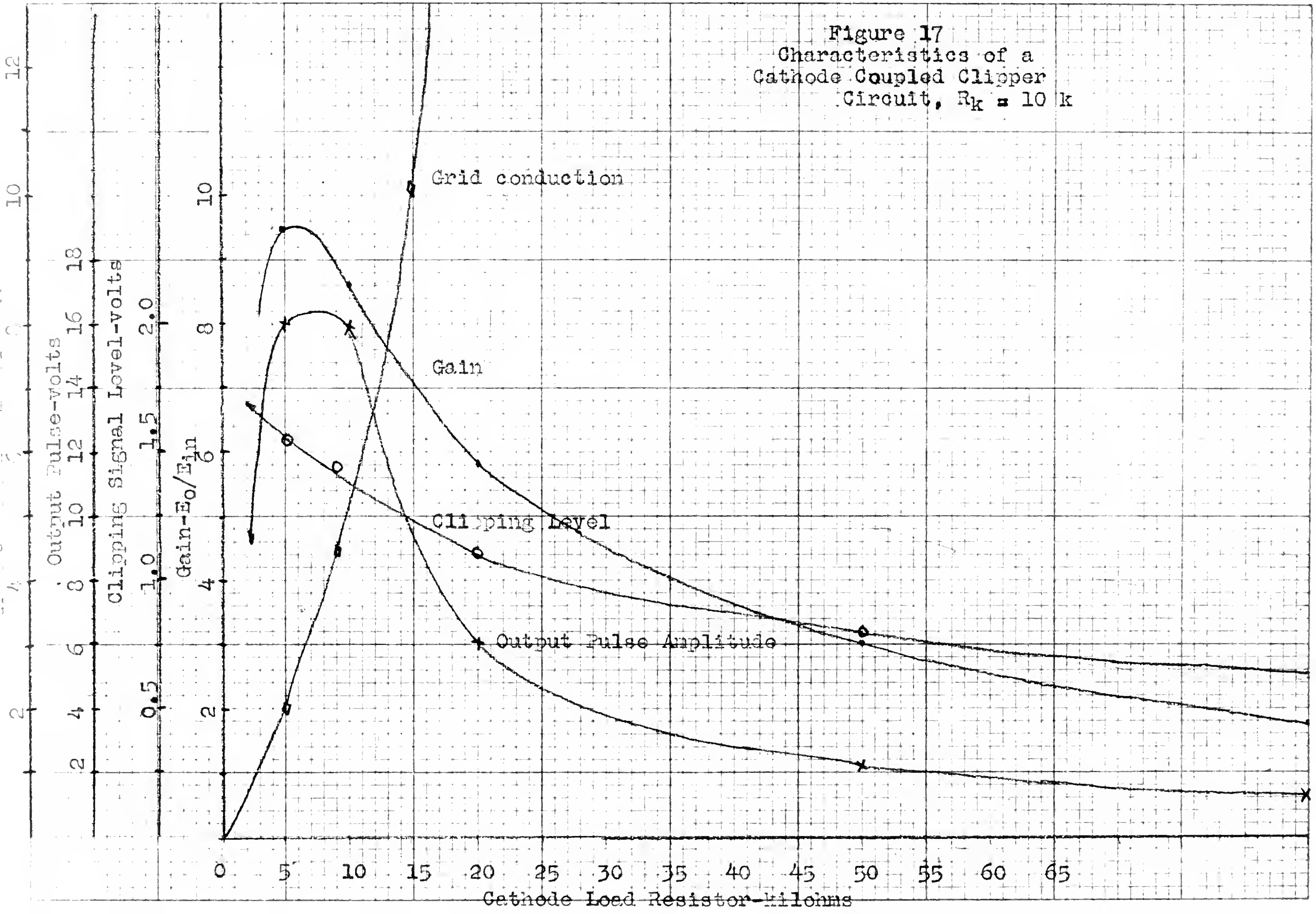


Figure 17
Characteristics of a
Cathode Coupled Clipper
Circuit, R_k = 10 k

it may be decided to sacrifice some of the clipping performance to obtain low level amplification in cascaded clipper stages. Clipping performance at high signal levels is not much affected by this. The value of the plate load resistor is a function of the upper signal frequency to be handled, and the output will be largely determined by the permissible clipper tube current. Figure 17 shows a typical set of performance curves for a clipper circuit using a 10k plate load resistor corresponding roughly to a upper frequency limit of 200 kcs.

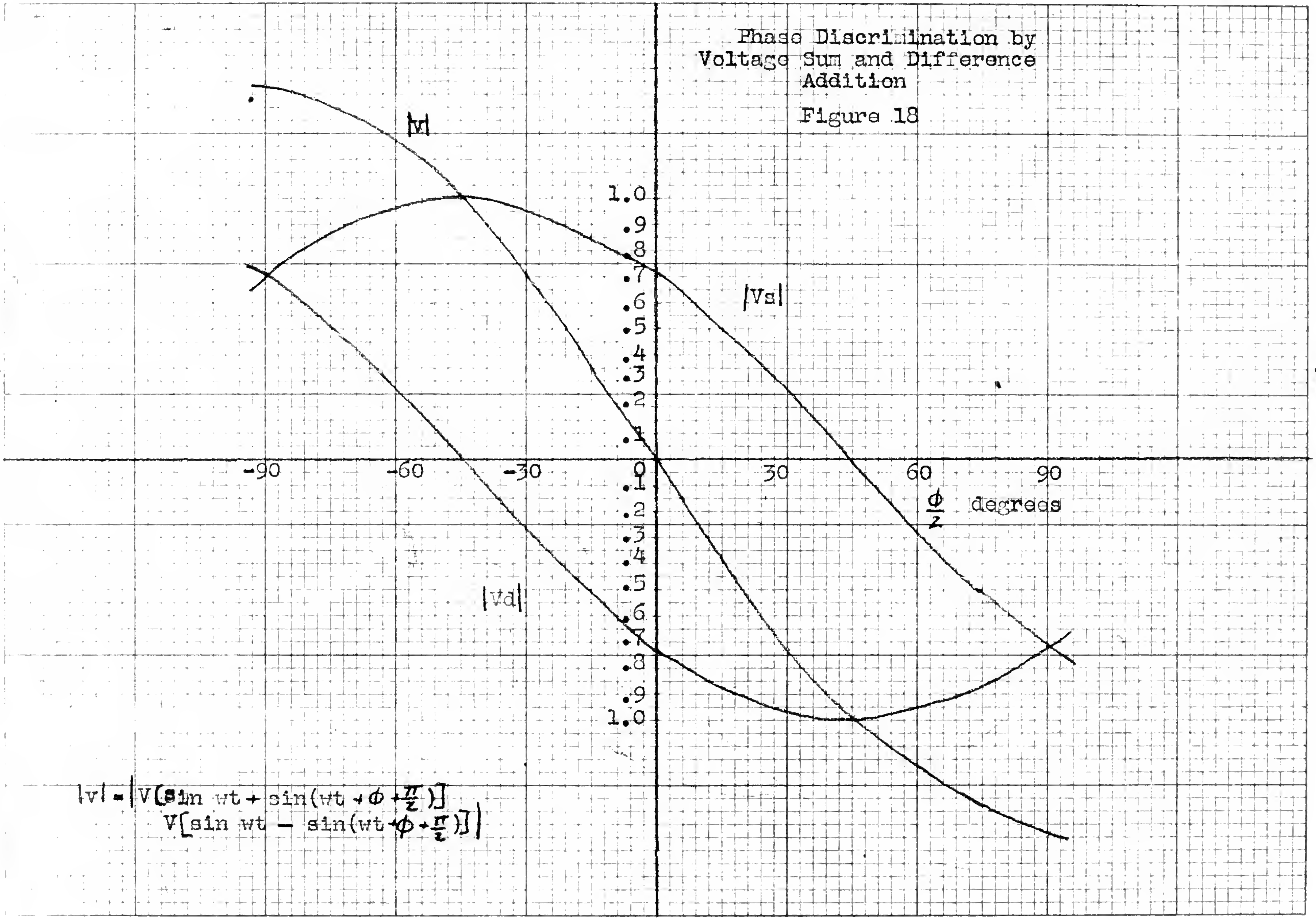
3. Phase Discriminators

The discriminator is actually a specialized member of the large family of circuits which are phase selective. The discriminator owes its position of relative importance to the fact that it is able to tell both phase reversal and magnitude of actual phase shift. Discriminators can be classified generally as belonging to one of two groups 1) wave summing circuits and 2) phase sensitive rectifiers. Both of these types discriminate by adding the signals, the sum or difference of the signals to one of the basic difference between the two is the sense in which the addition is performed.

In the nature of an introduction to the subject assume that the two signals to be compared are sinusoidal and that the discrimination is to be performed by adding the sum of the signals to the difference. This is the method employed by the well known Foster-Seely discriminator circuit and the power frequency phase sensitive rectifier widely used in servo systems.

If

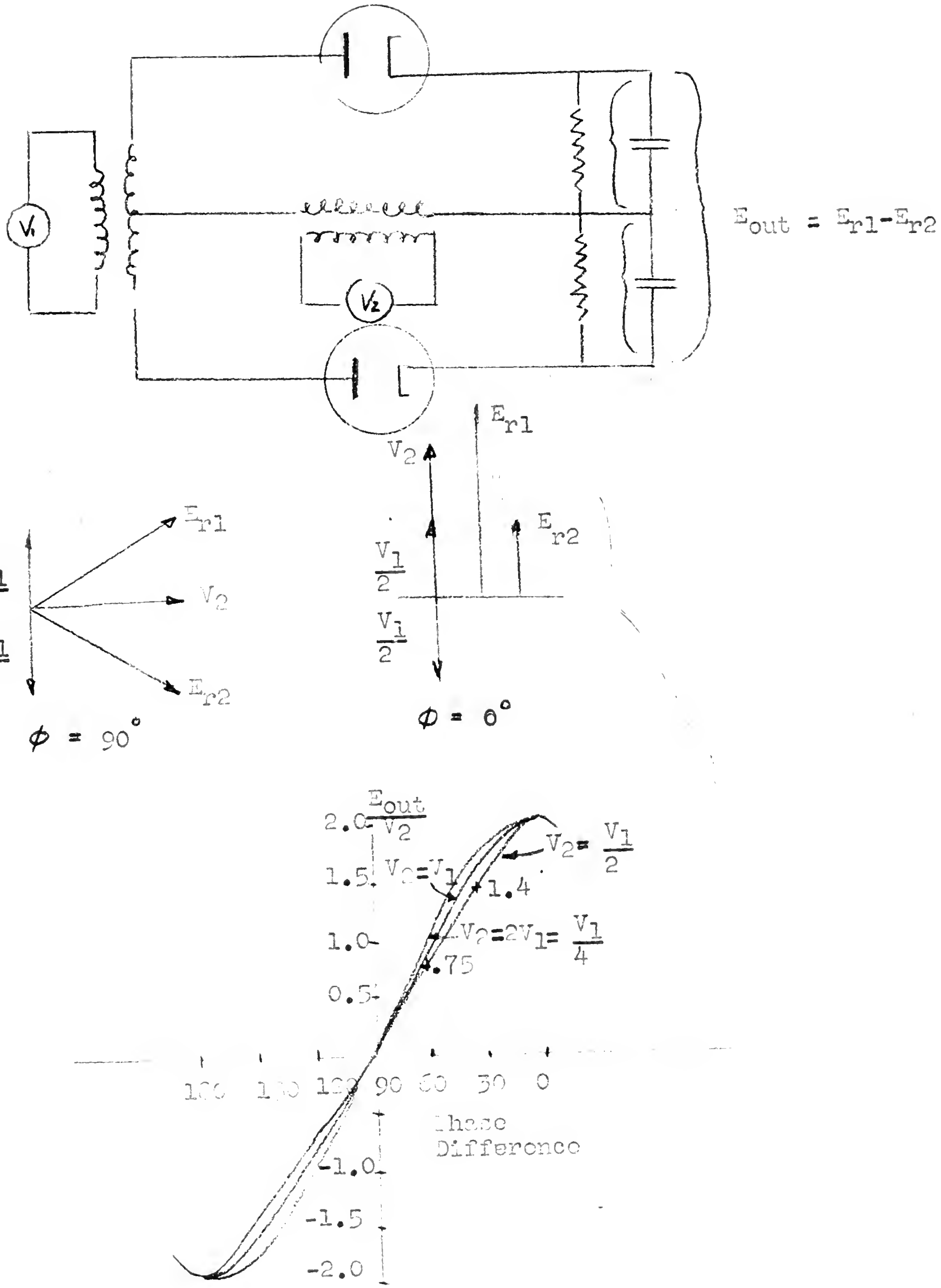
$$v_1 = V_1 \sin(\omega t)$$
$$v_2 = V_2 \sin(\omega t + \phi)$$



Phase Discrimination by
Voltage Sum and Difference
Addition

Figure 18

Figure 19
Output of a Peak Detecting Discriminator



Clearly the sum or difference of the two signals will vary as the amplitude of either one is varied, hence to render the output of the discriminator amplitude insensitive impose the condition that V_1 equal V_2.

Adding the two signals and reducing by trigonometric means,

$$\begin{aligned} V_1 + V_2 &= V[\sin(\omega t) + \sin(\omega t + \phi)] \\ &= V[\sin\{(\omega t + \tfrac{\phi}{2}) - \tfrac{\phi}{2}\} + \sin\{(\omega t + \tfrac{\phi}{2}) + \tfrac{\phi}{2}\}] \\ &= V[\sin(\omega t + \tfrac{\phi}{2})\cos\tfrac{\phi}{2} - \cos(\omega t + \tfrac{\phi}{2})\sin\tfrac{\phi}{2} \\ &\qquad + \cos(\omega t + \tfrac{\phi}{2})\sin\tfrac{\phi}{2} + \sin(\omega t + \tfrac{\phi}{2})\cos\tfrac{\phi}{2} \\ &= 2V\cos\tfrac{\phi}{2}\sin(\omega t + \tfrac{\phi}{2}) \end{aligned} \tag{2-8}$$

Similarly

$$V_1 - V_2 = -2V\sin\tfrac{\phi}{2}\cos(\omega t + \tfrac{\phi}{2}) \tag{2-9}$$

Now if a phase difference of 90 degrees is inserted in series with V2 prior to addition and subtraction

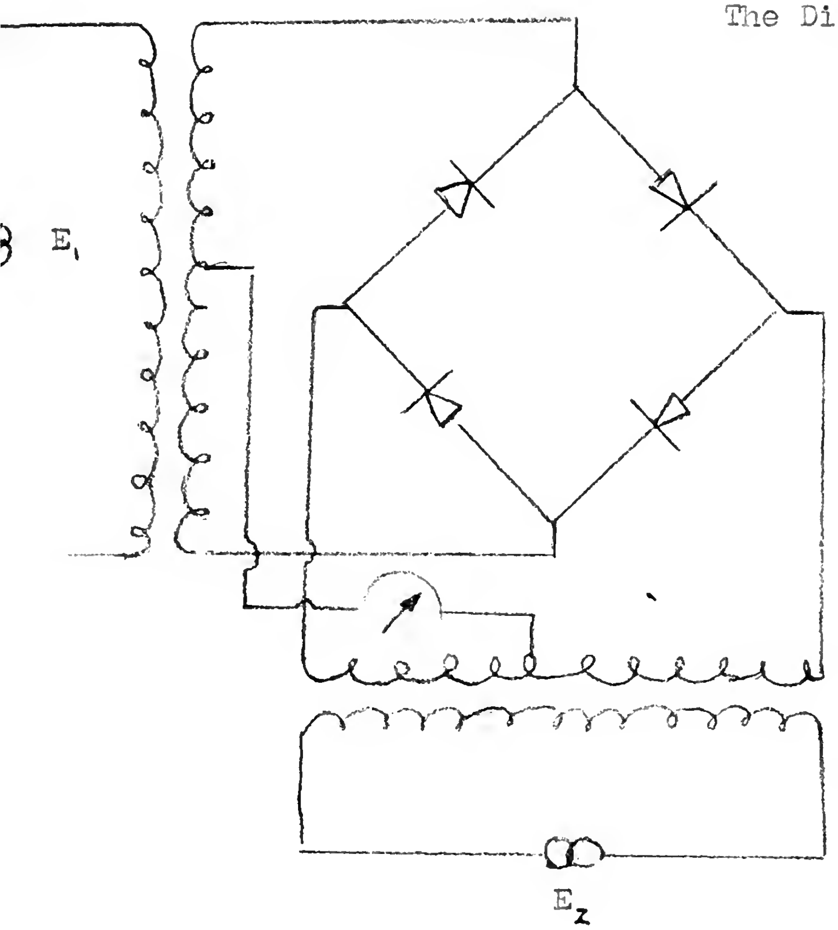
$$V_S = V_1 + V_2 = 2V\cos(\tfrac{\phi}{2} + \tfrac{\pi}{4})\sin(\omega t + \tfrac{\phi}{2} + \tfrac{\pi}{4}) \tag{2-10}$$

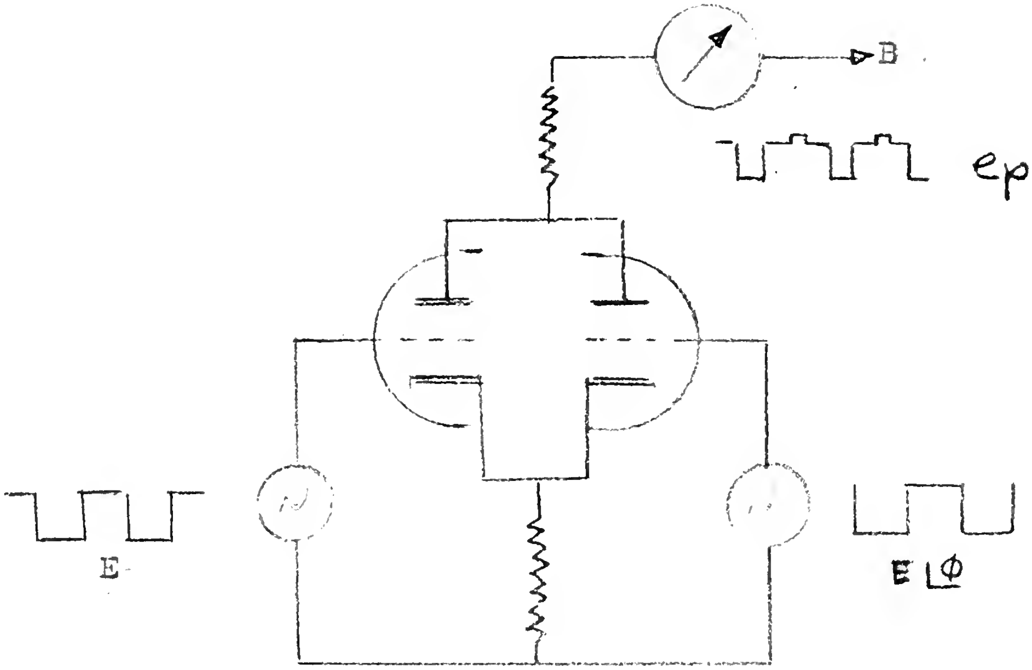
$$V_D = V_1 - V_2 = -2V\sin(\tfrac{\phi}{2} + \tfrac{\pi}{4})\cos(\omega t + \tfrac{\phi}{2} + \tfrac{\pi}{4}) \tag{2-11}$$

Discounting the time varying term and considering Vs plus Vd as a possible measure of the phase relationship, Figure 18, it is observed that Vs plus Vd is a sinusoidal function having its minimum value for Ø equal zero. Identical results can be arrived at by assuming the 90 degree phase shift in series with V1. Since Vs plus Vd without the shift is a maximum for equal zero, the 90 degree shift provides for a function which has its maximum rate of change at the condition of phase coincidence, a desirable

Figure 20

The Diode Bridge or Ring Modulator





A Vacuum Tube Ci~~b~~rcuit Having Properties Similar to the Diode Bridge

feature.

The well known Foster Seely discriminator is illustrated in Figure 19. The peak amplitudes of V2 and V1/2 are added and subtracted then the sum and differences detected and added together in the output of the circuit. If V2 is not equal to V1 this output varies from a sinusoidal and approaches linearity for V2 equal V1/2.

The diode bridge, or ring modulator, is probably the most sensitive discriminator circuit. Objectionable features are that it must be transformer fed and that the diodes must fairly well matched. When the phase of one signal is first shifted 90 degrees this type of discriminator can give an unambiguous output from plus or minus 90 degrees of phase shift, Figure 20. To verify this statement consider that the indicating device is a center scale, average reading milliammeter, and that the signal sources are in phase. By tracing out the current flow it is seen that its direction through the meter is always in one direction only giving maximum deflection in one direction. At 90 degrees of phase difference the average current through the meter is zero, and at 180 degrees the current is maximum in the reverse direction. Furthermore, if the signal waveforms are square and of equal amplitude the average current is a linear function of the phase difference.

In the use of this circuit damping resistors across the transformer secondaries are a necessity. They should be of small tolerance. Trouble may also be encountered in dissimilarity of the diodes in which case precision resistors across the diodes to degenerate the discrepancies in back resistance may prove helpful. Useful information as to the details

of diode selection and procedure in system application of this circuit is to be found in literature on speech equipment used for telephone and single sideband radio transmission.

The transformer required is an expensive item since it must necessarily have a fairly (minus 3 db.) flat response. If, for example, the signal sources were square waves of frequency from 30 to 10,000 cycles the transformer would have to be flat from 3 cycles to 100 kcs.

For direct addition of signal voltages the basic vacuum tube circuit is a common plate or common cathode summing chain. To a first approximation which is sufficiently accurate for most engineering applications

(Plate) $$\frac{1}{R_\ell}\sum_n e_n = -\frac{e_{(sum)}}{K}\,\frac{n+1}{R_\ell} - \frac{e_{(sum)}}{R_e}\ ;\quad e_{sum} \doteq -\sum_n e_n \qquad (2\text{-}12)$$

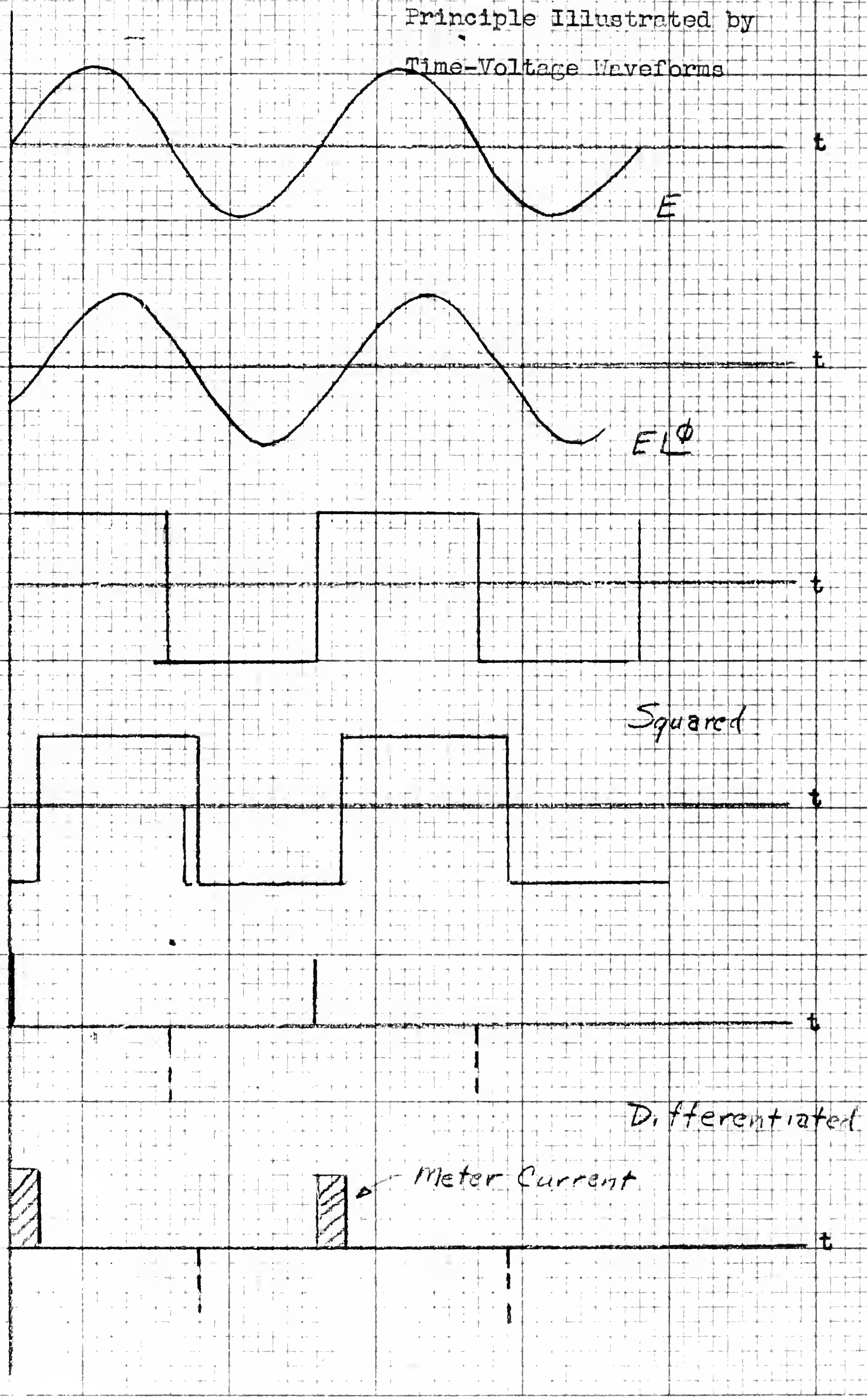
and

(Cathode) $$e_{(sum)} = \frac{\mu Y_p}{n(\mu+1)Y_p + Y_k}\sum_n e_n\ ;\quad e_{(sum)} \doteq \frac{1}{n}\sum_n e_n \qquad (2\text{-}13)$$

When two sinusoidal voltages of the same frequency are added the resultant is another sinusoidal of the same frequency and of amplitude determined by the relative amplitudes and phases of the two signal voltages. If the amplitudes can be held invariant, the output sum voltage becomes a measure of the relative phase. A phasemeter employing this principle of discrimination will, in the absence of any other operations performed, have an output which is sinusoidal and of zero magnitude when the phase difference is 180 degrees. Maximum output and minimum sensitivity to phase difference occurs at phase coincidence which is considered to be undesirable since, a priori, it would seem the most important observation to be made is of

Figure 21

Technology Institute Phasemeter Principle Illustrated by Time-Voltage Waveforms



the similarity of two characteristics rather than the dissimilarity.

Florman's phase meter[5] employs this method together with the basic principle of the Technology Institute Phasemeter to resolve ambiguity about 90 and 270 degrees. This last instrument is possibly the only truly portable instrument in anything resembling common use today and employs a unique principle worthy of some discussion.

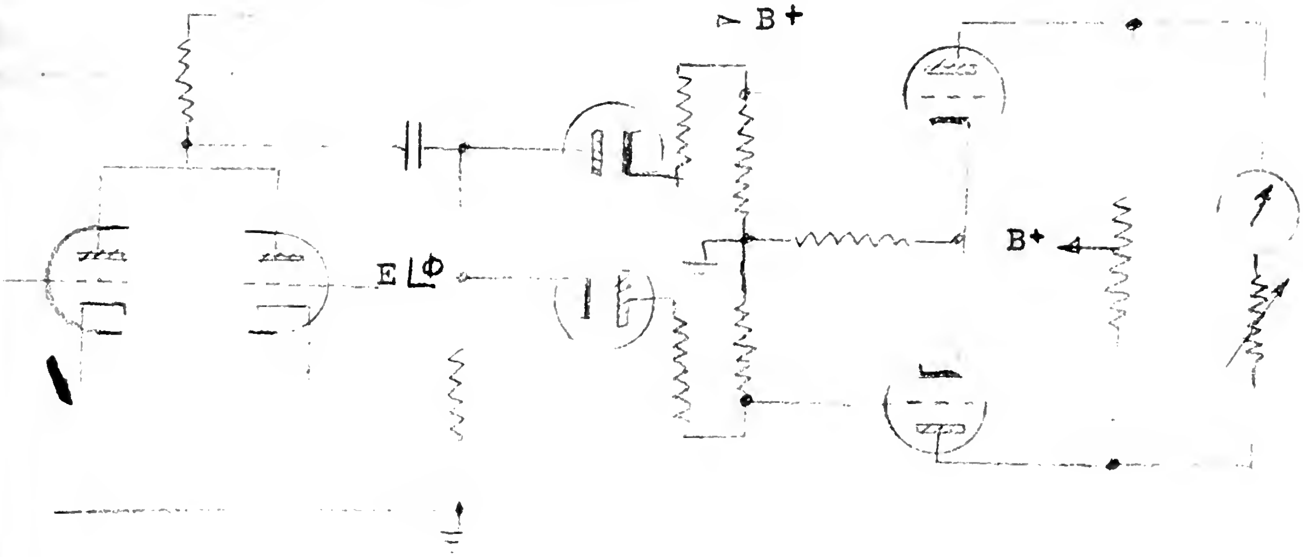
The Technology Institute Phasemeter[9] can be thought of as a summing device in that it adds the on-time of a bistable circuit to the off-time. The reference and measured signals are run through a series of amplifiers and cathode coupled clipper circuits to provide an essentially square wave. These signals are differentiated and the resulting voltage pulses used to trigger an Eccles-Jordan circuit from one state to the other. The indicating device is an average reading milliammeter inserted in the cathode circuit of one on the tubes in the Eccles-Jordan circuit. Provision must be made to insure that the bi-stable circuit will return to one particular condition in the absence of signals. This principle is illustrated in Figure 21.

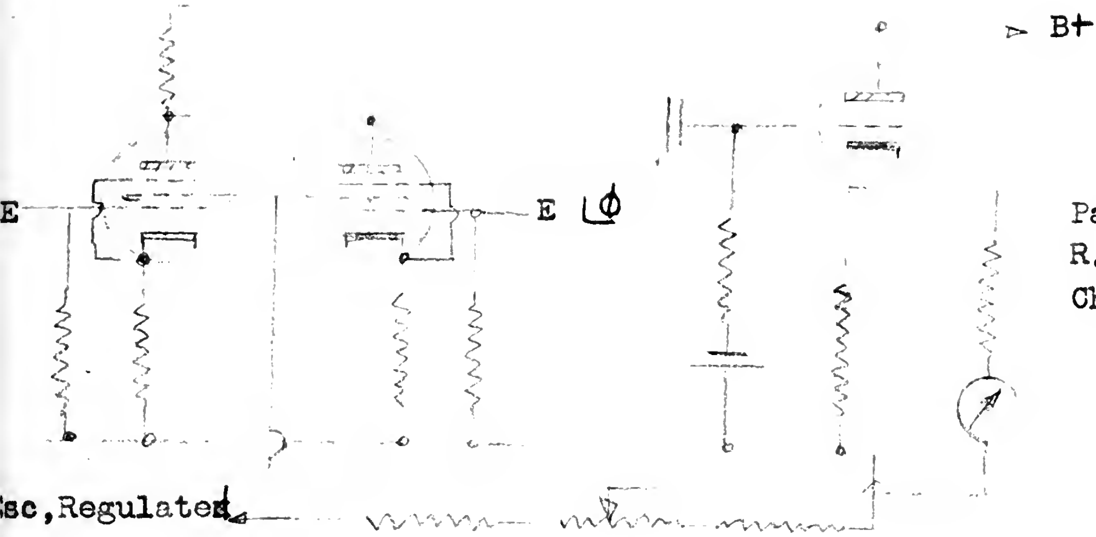
The phasemeter of Krause and Watson[8] gets around the 180 degree null condition by use of a cathode follower in one channel to insert an additional 180 degree shift. The effect on the summing amplifier of amplitude changes due to insertion of this cathode follower is open to conjecture.

The very useful circuit of Figure[20] can provide an output which is a linear function of phase difference of the input signals when these signals are square waves. This makes the circuit posses identical performance characteristics as the diode bridge for similar signal

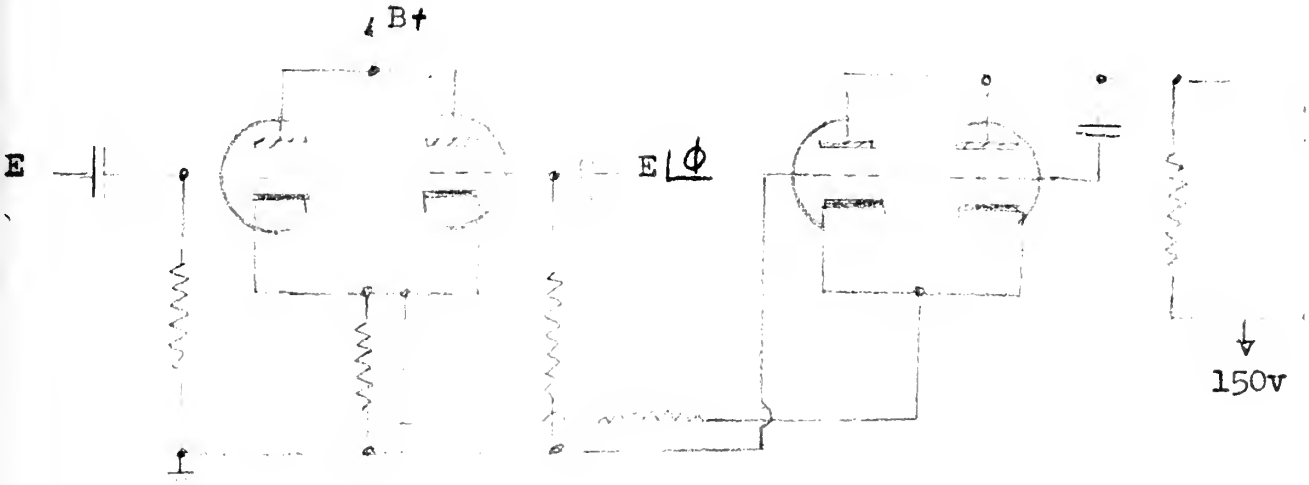
Figure 22

Some Typical Summing Amplifiers



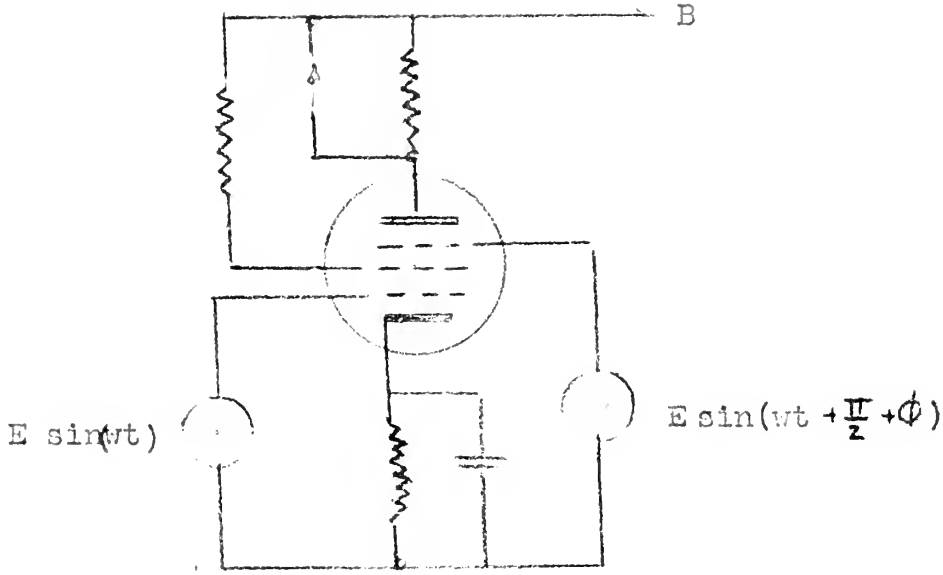


Patent Pending
R.V.Werner, assigne
Chance-Vought A.C.



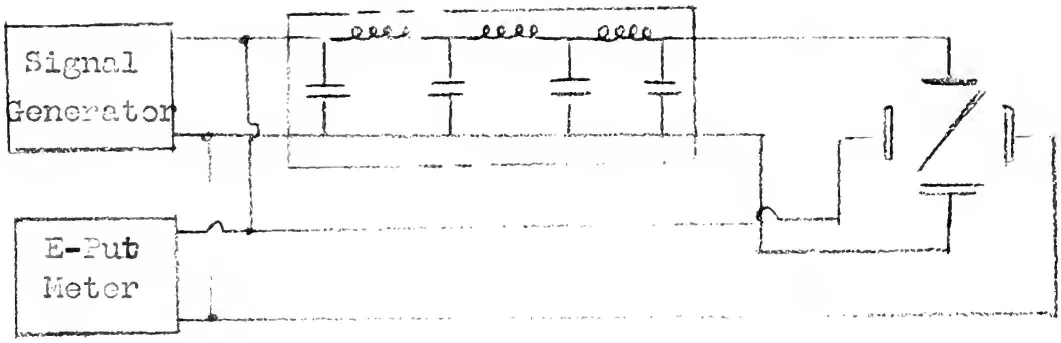
Patented, W.F. Davis, assigned Chance-Vought A.C.

Figure 23



A Phase Sensitive Circuit Using a Dual Control Tube

Figure 24



Method of Obtaining the Base Frequency for a Standard Lag Line

sources. This is accomplished by choosing the cathode resistor to be of such magnitude as to degenerate the gain of each section to one half value when signals are applied to both grids.

A large number of practical summing amplifiers are currently in use. Several particular circuit configurations are shown in Figure 22 for purposes of illustration. In each case the essential difference is only in the metering circuit, a basic common plate or common cathode summing chain being used for addition of signal voltages.

The first circuit employs full wave rectification and a push-pull triode amplifier to isolate the meter circuit from the summing circuit and amplify the output of the summing amplifier. The second circuit uses a cathode follower in the conventional manner to isolate the metering circuit. In the third circuit the metering cathode follower is normally cut off. With the application of signal voltages decrease in current through the cathode summing resistor permits current to flow in pulses through the metering circuit. Positive feedback in the second section of the metering tube insures rapid conformance of the metering circuit to changes in signal phase.

If the signal amplitudes are large enough to cut off a dual control tube such as a 6AS6 or 6BN6 the tube can be used as a summing amplifier type discriminator directly. A milliammeter in the plate circuit or voltmeter across the plate load serves as an indicator, Figure 23. For large grid signals these tubes are saturated and cut off rapidly by the signals at either grid and the plate wave form is essentially that shown for Figure 20. The big problem is to feed large signals into the grids of these tubes without getting into trouble with grid current. If the

signals are fed in through an R-C coupling circuit signal bias is built up. Cathode follower or transformer coupling is to be preferred. Also this trouble could be circumvented if the grid signals were square in from at the onset. In this manner the tube could be rapidly cut on and off without grid current flowing.

CHAPTER IV

STANDARDS FOR PHASE MEASUREMENT AND CALIBRATION

No primary standard for phase measurement is available. That is it is not possible to relate the property to any of the canoniacal qualtities directly such as current measurement by Faraday's electrolysis method or voltage measurement by standard Weston cell. This would seem to follow from the fact that while it is possible to note the passage of time in a very accurate manner it is not possible to advance or retard time by an arbitrary amount.

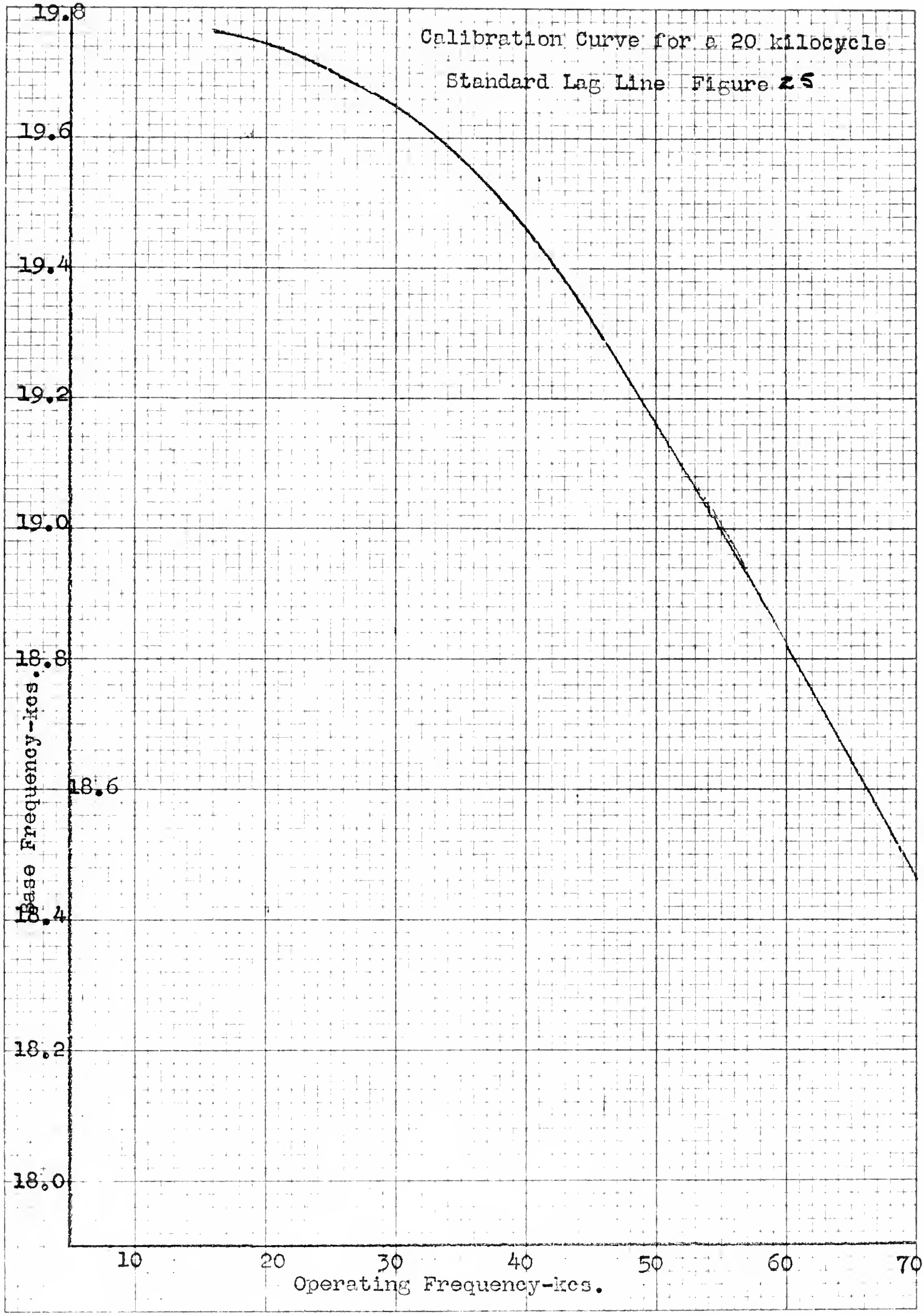
Two highly precise secondary standards will be discussed. They are 1) an artificial transmission line which can be accurately calibrated at one or more frequencies for use at any frequency within the limits of its construction and 2) a phase shifter of the goniometer type which can be made highly accurate for one given frequency. It is felt that these two could satisfy any need for a phase standard that might arise.

1. A Lag-line Standard.

A simple lumped constant multi-section transmission line can be built to any electrical length desired. As an arbitrary criterion say the line is to be 360 electrical degrees long at the lowest frequency to be encountered. The line is a multisection, constant-k filter which obeys the well known laws for a network of this type.

$$\tau = \sqrt{\ell c} \; ; \qquad Z_0 = \sqrt{\frac{\ell}{c}} \; ; \qquad f(\text{cutoff}) = \frac{1}{\pi \tau}$$

A jig and CRO arrangement described in Appendix I can be used to assure that the sections are identical.


Calibration Curve for a 20 kilocycle
Standard Lag Line Figure 25
19.8
19.6
19.4
19.2
19.0
18.8
18.6
18.4
18.2
18.0
Base Frequency-kcs.
10
20
30
40
50
60
70
Operating Frequency-kcs.

Once the line has been constructed it is only necessary to calibrate its electrical length by use of a CRO and signal generator. (Figure 24) A calibration chart is then constructed for use in the frequency range using as one argument a "base" frequency at which the electrical length is an integral multiple of π radians. The phase lag is then simply related to the base frequency by the equation

$$\theta_{\bullet} = \frac{f}{f_0}\,\theta_0 \qquad (\theta_0 \text{ is electrical length at } f_0) \qquad (3\text{-}1)$$

A typical curve for a 20 kcs standard line having an electrical length of 360 degrees is shown in Figure 25.

The recommended construction of this line is with use of inductances of litz wire over powdered iron torroidal cores and silver mica condensors.[19]

2. A Goniometer Phase Standard

An extremely accurate phase standard of the goniometer type can be constructed. This method of phase control is well known and one of its most important uses which comes to mind is in the Meacham range unit widely used in radar. However, the goniometer capacitor used in the Meacham unit is expensive and since the principle can be adapted to use with either resistance or inductive elements a resistance goniometer[21] would seem to be more attractive for most engineering uses, Figure 26.

There is an inherent error in this device due to the fact that there is a circulating current in the resistance elements of a round potentiometer and an additional current in the slider circuit. If the slider current can be neglected the phase angle error can be obtained by graphical solution of the equation

Resistance Goniometer and Typical
Quadrature Voltage Generator Circuit

Figure 26

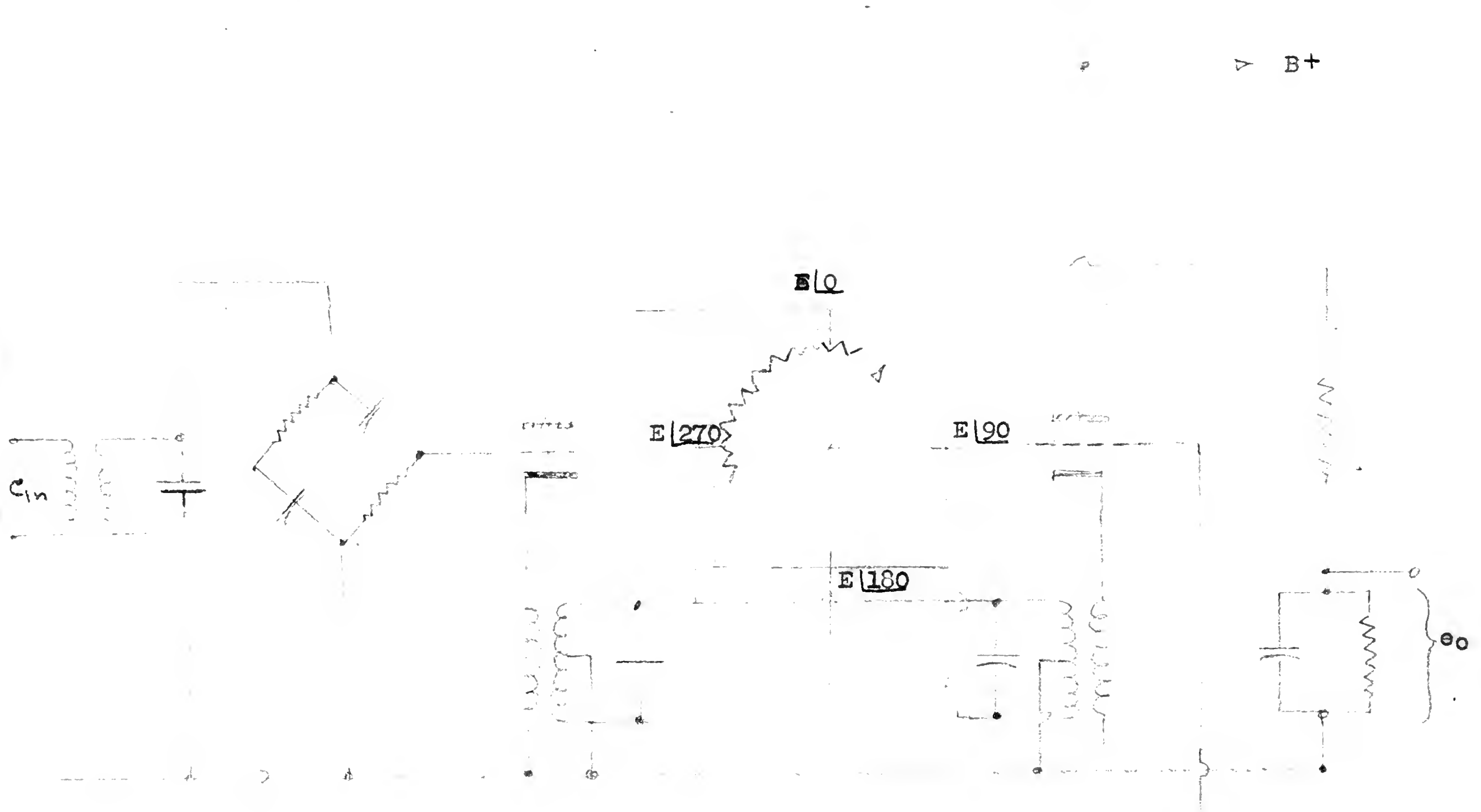


Figure 26

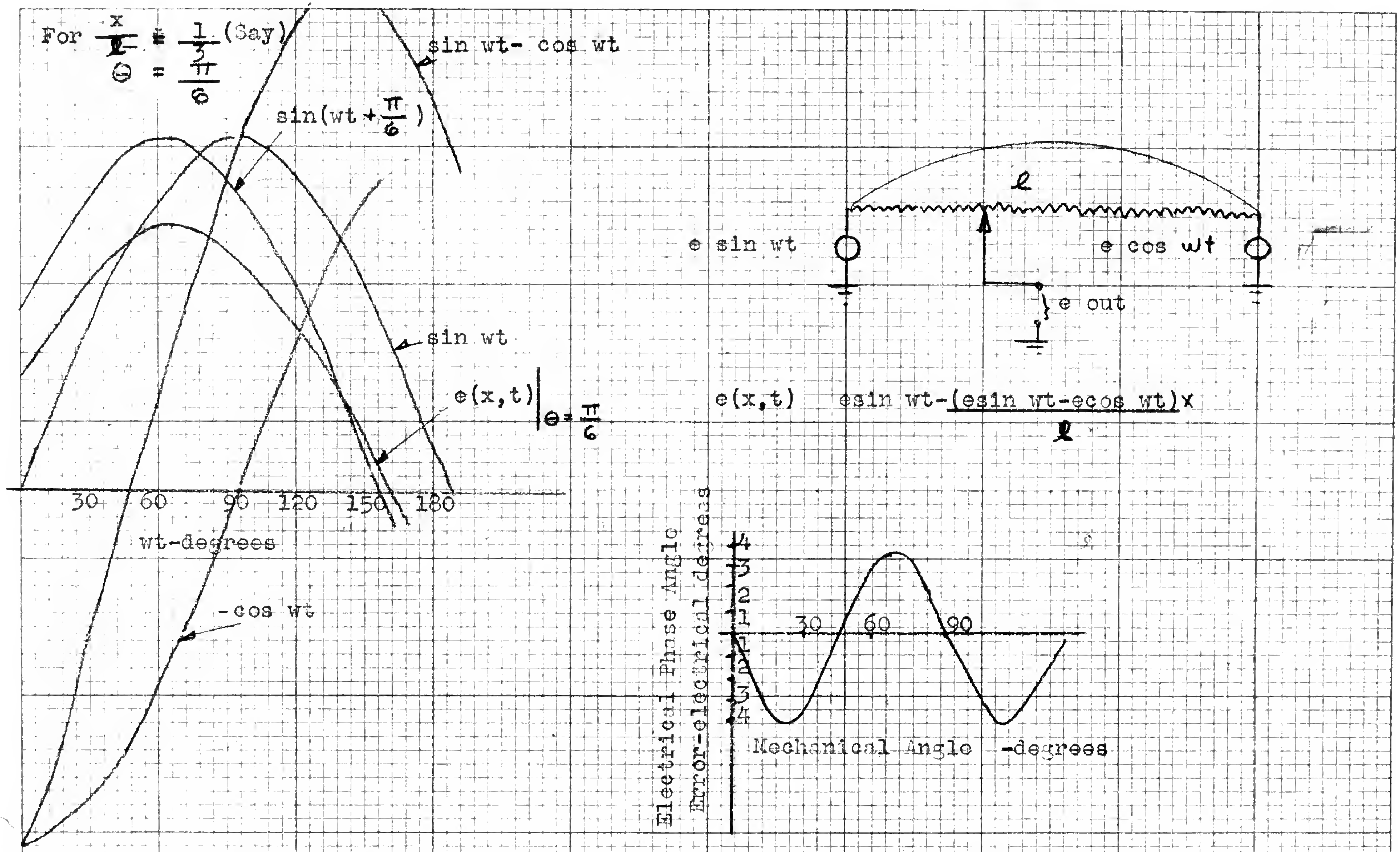


Figure 27

Resistance Goniometer
Output Phase Error

$$x = \frac{\theta(2l)}{\pi}$$

$$\mathcal{E}(x) = E \sin\left(\omega t + \frac{\pi x}{2l}\right) - e\left[\sin(\omega t) - \frac{x}{l}(\sin(\omega t) - \cos(\omega t))\right] \tag{3-2}$$

where x is the distance along the potentiometer from a tap and l is the distance between two taps. A curve of phase error versus mechanical angle θ is shown in Figure 27. This error can be reduced to a very small amount by appropriate frequency division.

It may be shown that if the potentiometer is made square and if the slider still moves in an arc the error becomes zero.

The idea of a resistance goniometer is encouraged by the fact that there are commercially available very accurate multiturn, linear potentiometer (Helipots). Unfortunately most of these potentiometers have characteristics resembling transmission lines at frequencies above power frequencies and the resistance looking back from the taps should be as high as possible to avoid reflections and discontinuities on this equivalent line. No theoretical analysis is possible becuase the exact equivalent circuit of the multiturn potentiometer is not known to a sufficiently accurate degree at high frequencies.

Inductance goniometers are also used in a number of applications, notably in radio direction finding equipment. The general development of their circuit follows the same line of reasoning as the resistance goniometer and the capacitance goniometer to be discussed later. If a choice is possible, the capacitance goniometer is to be preferred since the coupling coefficients of the coils cannot be maintained with the tolerances possible in the capacitance device.

Capacitance Goniometer and
Equivalent Circuit

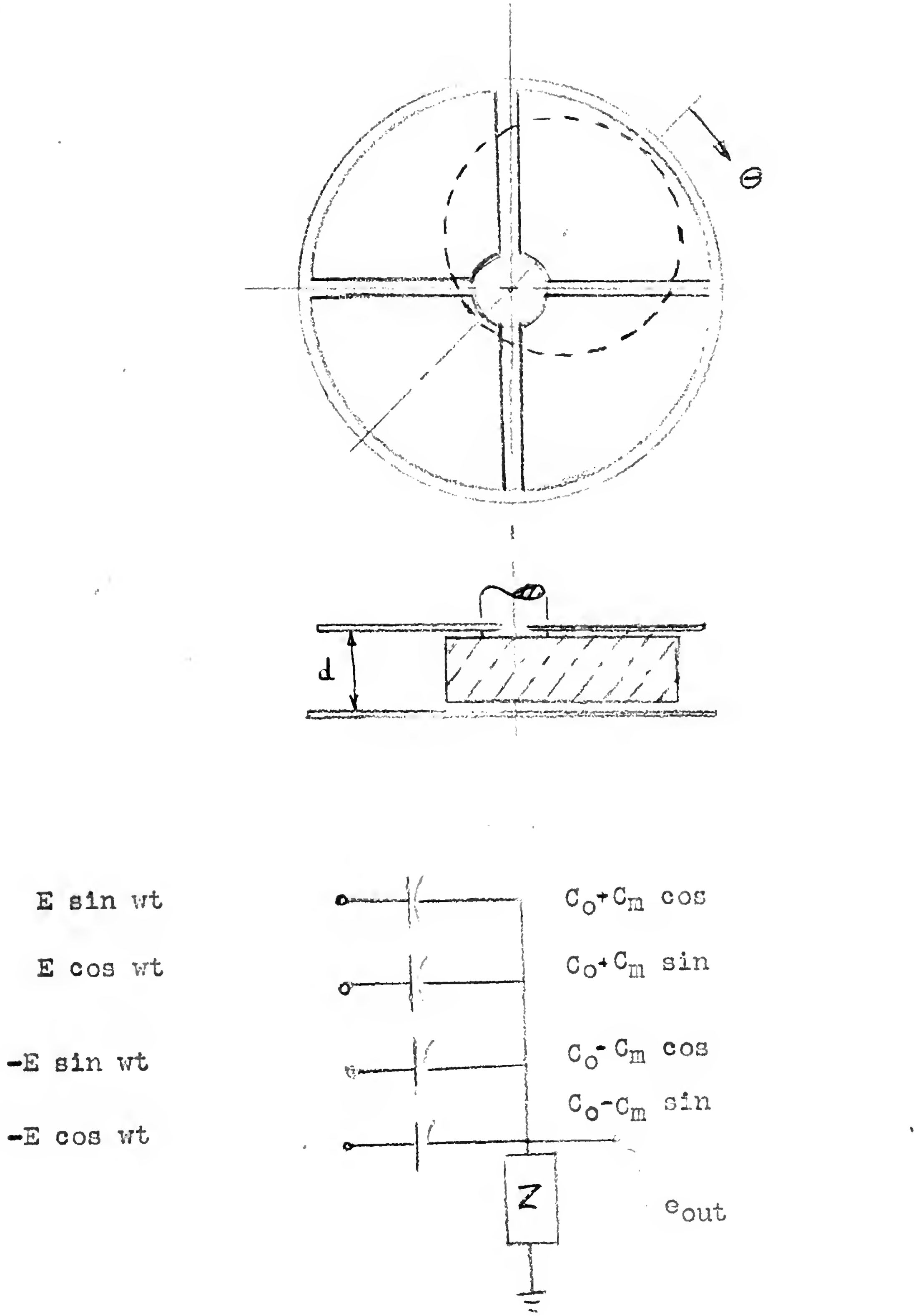


Figure 28

The capacitance goniometer has had notable success in radar and omnirange beacon circuits, however, the precision capacitor is an expensive item. In the figure 28 the capacitance of the individual sections is

$$C = \frac{kA}{d} = \frac{k(\text{Air}) \times \text{Area}(\text{Air})}{d} + \frac{k(\text{Diel.}) \times A(\text{Diel.})}{d} \qquad (3\text{-}3)$$

The dielectric constant is made to vary sinusoidally with the angle of mechanical rotation θ , that is

$$C(\text{Section}) = C_o + C_m \sin\theta$$

Then by writing the sums of the branch currents as the current through the load impedance and solving for the output voltage, it is seen that

$$e_{(out)} = \frac{2E_m}{X(C_m)} \sin(\omega t + \theta) + \text{D.C. Term.} \qquad (3\text{-}4)$$

The electrical phase angle of the output signal has been made to vary directly as the mechanical angle.

CHAPTER V

CONCLUSIONS

The problem of phase measurement and control is of great importance to the armed forces and industry alike. A great deal of work has been done and much is now going on, a large portion of which must be assumed to be guarded by classification. Ultimately the quality of phase may assume the same importance in the electronic field as is now held by frequency. To this end and solely on the basis of the general unclassified matter presented in this paper the following conclusions are drawn.

1) There exists a need for a broad and phase shift network which is not as critically dependent upon component values as those presently available.

2) The range of frequencies of the existing phase shift networks should be extended.

3) A precision phase shifter which is frequency insensitive would be the ideal method to employ in the measurement of phase. In this way every value of shift could be obtained by the null method which is inherently the most accurate means of electrical measurement available.

4) A theoretical study as to whether phase discrimination is most accurately and economically done at microwave frequencies with magic T's and rat-races or at some intermediate frequency after heterodyning might be useful.

5) A reliable commercial phase meter with an accuracy of plus or minus 1 degree and a range of 0.1 to 1000 kilocycles obtained with a reasonable number of controls should prove a marketable item.

6) It seems unlikely any new circuits or techniques for the measurement or control of phase will be forthcoming. Any improvement must be by refinement of existing circuitry or different application of existing techniques.

7) As a precision broadband phase measuring device the instrument outlined in Figure 29 is suggested. Quardrature voltages for a precision goniometer of either the resistance or capacitance type are obtained from broadband phase shift networks. The goniometer is used to insert a precision phase shift in series with one of the signals to be compared to bring about phase coincidence. The phase null indicator is a diode bridge circuit since this circuit will suitable as a phase null indicator with complex waveforms. Amplitude variations are removed by a series of clipper circuits.

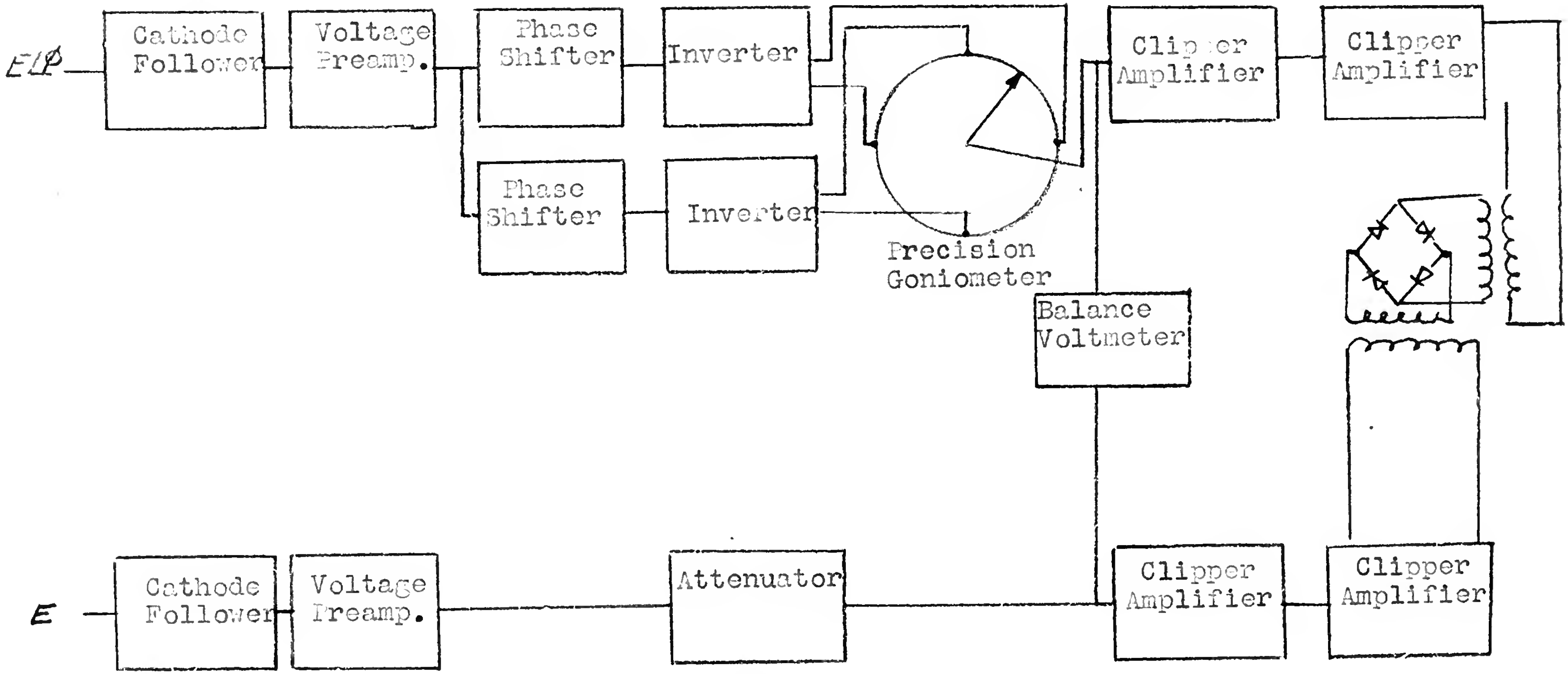


Figure 29

Proposed Precision Phasemeter

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APPENDIX I

A Precision Measurement of Phase Coincidence Using a Cathode Ray Oscilloscope

This method of obtaining precise measurements around phase coincidence is abstracted from a method of production testing in use by a large electronics concern. Because of the numerous incidences encountered in collecting the bibliographical material for this thesis where this method has been used as a laboratory procedure it is considered sufficiently important to be included as an appendix.

Given the problem of measuring the exact condition of phase coincidence or slight errors therefrom:

1. A CRO is reworked to provide a high degree of linearity and similarity in its deflection amplifiers as evidenced by standard tests performed on these amplifiers.

2. The two signals to be compared in phase are applied to the vertical and horizontal deflection amplifiers respectively producing the straight line pattern characteristic of this condition.

3. The signal levels are recorded and each increased by a like factor. This puts the pattern off the scope face, but considering the deflection to be still linear the maximum trace excursion can still be considered a being the original excursion multiplied by the voltage factor.

4. Small vertical and horizontal intercepts indicating departures from phase coincidence previously not observable can now be measured.

APPENDIX II

Phase Invariance During the Heterodyning Process

For the general case of plate current as a function of grid voltage

$$i_p = g_m\left(e_g + \frac{e_b}{\mu}\right) + \frac{1}{2!}\frac{\partial g_m}{\partial e_g}\left(e_g + \frac{e_b}{\mu}\right)^2 + \frac{\partial^2 g_m}{\partial e_g^2}\frac{1}{3!}\left(e_g + \frac{e_b}{\mu}\right)^3 + \text{etc}$$

If, e_b can be described as having a constant relationship to which amounts to saying that μ remains constant the expression becomes

$$i_p = g_m k e_g + \frac{\partial g_m}{\partial e_g}\frac{k^2}{2!}e_g^2 + \frac{\partial^2 g_m}{\partial e_g^2}\frac{k^3}{3!}e_g^3 + \cdots \text{etc.}$$

and the variation of with is normally sufficiently small to make the larger power terms vanishingly small. This expression shows that there is a value of plate current which is proportional to the square of the grid voltage. Square law conversion, as it is called, is more efficiently accomplished by choosing proper circuit parameters to produce a parabolic transfer characteristic from which the expression for plate current become directly

$$i_p = a e_g^2$$

By applying the usual trignometric reduction formulas to

$$i_p = a\left[A\cos(\omega_c t + \phi) + B\cos(\omega_o t)\right] \qquad \omega_c > \omega_o$$

an expression for the different frequency components is obtained

$$i_p = \frac{aA^2}{2} + \frac{aA^2}{2}\cos 2(\omega_c t + \phi) + 2\frac{aAB}{2}\cos(\omega_c t + \phi + \omega_o t)$$
$$+ 2\frac{aAB}{2}(\omega_c t + \phi - \omega_o t) + \frac{aB^2}{2}\cos(2\omega_o t) + \frac{aB^2}{2}$$

If sufficiently selective circuits are postulated to remove the unwanted frequency components, the desired intermediate frequency is

$$aAB \cos[(\omega_c - \omega_o)t + \phi]$$

In the general conversion case where the two signals may be coupled in the electron beam it is customary to postulate that the local oscillator amplitude is very much greater than the signal amplitude and that the tube transconductance becomes a periodically varying function of the local oscillator signal amplitude. By expressing transconductance as a Fourier Series

$$g_m = \frac{b_o}{2} + b_1 \cos \omega_o t + b_2 \cos 2\omega_o t + \cdots\cdots$$

and the a-c plate current due to the signal voltage is

$$i_p = g_m E_{sig} \cos(\omega_c t + \phi)$$

Again removing all unwanted higher and lower frequency terms by use of selective circuits the desired intermediate frequency becomes

$$i_p = b_1 E_{sig} \cos(\omega_c t + \phi) \cos \omega_o t$$

$$= \frac{b_1 E_{sig}}{2} \cos[(\omega_c - \omega_o)t + \phi] + \cdots\cdots$$

APPENDIX III

Analysis of the Cathode Coupled Clipper Circuit

This circuit is illustrated in Figure 16. Complete development of this circuit was originally done as a government sponsored research project and is completely discussed in reference 6 . Only the portions of that article describing the action of the basic circuit as a clipper are treated here. The concept of negative resistance is analogous to positive feedback and, as might be suspected when the concept is evolved out of gain considerations, positive feedback can affect the operation as a clipper. The effects of this type of feedback are treated in the original article.

It is often the practice in the use of this circuit to cascade several slipping stages, perhaps with interspersed amplification stages, when the available signal is very low and very steep wave fronts are desired. In this case any additional gain available in the clipper stages can be used. To this end the balance of this appendix is devoted.

The circuit can be thought of as a cathode follower, the output of which drives a plate load triode amplifier by grid injection. The first stage is cut off by negative gird signal. Since the grid of the triode amplifier is at a.c. ground and fixed d.c. potential a positive going grid signal soon raises the cathode voltage and the second tube is cut off. For negative going grid signal the second tube hastens the time of cut off by tending to keep the common cathode at a high potential.

If the upper and lower voltage excursions are designated e (+) and e (-) the following relationships can be developed for the clipped wave, Figure 16.

Time A-B $= T = 1/f = 2\pi/\omega$

$$e(+) = E_{sm} \sin \omega t \doteq E_{sm}\,\omega t \qquad t_1 \le t \le t_2$$

Therefore in the time interval a-b the instantaneous voltage is, since the sine of the angle is practically equal to the angle

$$e(+) = E_{sm}\,\omega t + E_{cc_1} \;;\; e(-) = -E_{sm}\,\omega t + E_{cc_1}$$

$$e(+) - e(-) = E_{sm}(t_2 - t_1)\,\omega = E_{sm}\,t\omega$$

$$\%\ Rise\ Time = t/T \times 100 \quad ; \quad t = \frac{e(+) - e(-)}{E_{sm}\,\omega}$$

$$= \frac{\dfrac{e(+) - e(-)}{E_{sm}\,\omega}}{2\pi/\omega} \times 100 = \frac{e(+) - e(-)}{2\pi E_{sm}} \times 100$$

The smaller the rise time the less the possibilities dissymmetry. If a tube parameter μ' which is defined as the ratio of plate voltage to grid voltage at plate cut off is introduced the values of e (+) and e (-) become

$$e(+) \cong E_{cc_2} + E_{bb}\,\frac{(\mu - \mu')}{\mu\mu'}$$

$$e(-) \cong E_{cc_2} - E_{bb}\,\frac{(\mu - \mu')}{\mu\mu'}$$

The figure of merit of this circuit is the "input ratio". This is the ratio of the maximum grid to ground signal which may be applied without causing grid current to flow to the minimum grid to ground signal at which clipping just begins.

$$\text{Input Ratio (I.R.)} = \frac{e_{max} - E_{cc_1}}{e(+) - E_{cc_1}} = \frac{E_{sm}}{e(+) - E_{cc_1}}$$

$$= \frac{\mu\mu'}{E_{bb}(\mu-\mu')}\left[\frac{E_{bb}R_k}{R_k + r_p} + \frac{E_{bb}}{\mu} - \frac{E_o R_k}{R_e}\right]$$

[E_o is output amplitude (peak)]

The characteristics of the next stage determine the amplitude of the permissible signal which may be applied, that is the clipper output pulse. The total input-output capacity and the frequency requirements determine the size of the plate resistor in the manner of any other RC amplifier. With Re and Eo fixed the expression for Input Ratio is maximized to give the following values for Rk and Eccl 2. The two grid biases are chosen to be equal to prevent phase dissymmetry distortion discussed in the text.

$$R_k = \sqrt{\frac{E_{bb} R_e r_p}{E_o}} - r_p$$

$$E_{cc_1} = E_{cc_2} = \sqrt{\frac{E_{bb} E_o r_p}{R_e}} - \frac{E_{bb}}{\mu} - \frac{E_o r_p}{R_e}$$

The relationship of the input ration to the rise time is

$$\text{Rise time } (\%) = \frac{e(+) - e(-)}{2\pi E_{sm1}} \times 100$$

$$\text{Since } e(-) = 2E_{cc} - e(+)$$

$$\text{Rise time} = 100\left\{\frac{2e(+) - 2E_{cc}}{2\pi E_{sm}}\right\} = \frac{100}{\pi\left[\frac{E_{sm}}{e(+) - E_{cc}}\right]}$$

$$= 100/\pi(I.R.)$$

Which indicates that for optimum clipper action with minimum rise time maximum input ratio should be used.

The problem is now to reconcile performance as a clipper with best

performance as a linear amplifier. The Taylor series expansion for the incremental plate current is

$$i_p = g_m\left(e_g + \frac{e_b}{\mu}\right) + \frac{1}{2!}\frac{\partial g_m}{\partial e_g}\left(e_g + \frac{e_b}{\mu}\right)^2 + \frac{1}{3!}\frac{\partial^2 g_m}{\partial e_g^2}\left(e_g + \frac{e_b}{\mu}\right)^3 + \cdots$$

and to a first approximation

$$i_p = g_m e_g + \frac{e_b}{r_p}$$

The following relationships can be developed.

1. $e_{c_1} = E \sin \omega t - e_k$

2. $e_k = (i_{p_1} + i_{p_2}) R_k$

3. $e_{c_2} = -e_k$

4. $i_{p_1} = g_m e_{c_1} + \dfrac{e_{b_1}}{r_p} = g_m e_{c_1} - \dfrac{(i_{p_1} + i_{p_2}) R_k}{r_p}$

5. $i_{p_2} = g_m e_{c_2} + \dfrac{e_{b_2}}{r_p} = g_m e_{c_2} - \dfrac{(i_{p_1} + i_{p_2}) R_k}{r_p} - \dfrac{i_{p_2} R_e}{r_p}$

6. $e_{(out)} = i_{p_2} R_e$

Solving for the plate currents and output voltage

I $\quad i_{p_1} = g_m E \sin \omega t - g_m R_k (i_{p_1} + i_{p_2}) - \dfrac{(i_{p_1} + i_{p_2}) R_k}{r_p}$

II $\quad i_{p_2} = -g_m R_k (i_{p_1} + i_{p_2}) - \dfrac{(i_{p_1} + i_{p_2}) R_k}{r_p} - \dfrac{i_{p_2} R_e}{r_p}$

Eliminating ip_1 and ip_2 between these equations the expression for gain is obtained as a function only of the circuit parameters.

$$(1 + g_m R_k + R_k/r_p)\, i_{p_1} + (g_m R_k + R_k/r_p)\, i_{p_2} = g_m E \sin \omega t$$

$$(g_m R_k + R_k/r_p)\, i_{p_1} + (1 + g_m R_k + R_e/r_p)\, i_{p_2} = 0$$

$$i_{p_2} = -\frac{\left[g_m E \sin \omega t \,(g_m R_k + R_k/r_p)\right]}{\left[1 + g_m R_k + R_k/r_p\right]\left[1 + g_m R_k + R_e/r_p\right] - \left[g_m R_k + R_k/r_p\right]^2}$$

$$\frac{e_{out}}{E(\sin \omega t)} = -\frac{\left[g_m^2 R_e R_k + g_m R_k R_e/r_p\right] r_p^2}{\left[r_p^2 + (2\mu + 1) R_k r_p + (\mu + 1) R_e R_k - (\mu + 1) R_k^2 + r_p R_e\right]}$$

$$K = -\frac{(\mu+1)\mu R_k R_\ell}{\mu\left(\frac{r_p}{g_m} + \frac{R_\ell}{g_m}\right) + (2\mu+1) R_k r_p + (\mu+1) R_\ell R_k - (\mu+1) R_k^2}$$

$$\mu + 1 \doteq \mu \quad ; \quad 2\mu + 1 \doteq 2\mu$$

$$K = -\frac{\mu R_k R_\ell}{\left(\frac{r_p}{g_m} + \frac{R_\ell}{g_m}\right) + 2 R_k r_p + R_\ell R_k - R_k^2}$$

If an attempt is made to maximize this gain expression for Rk

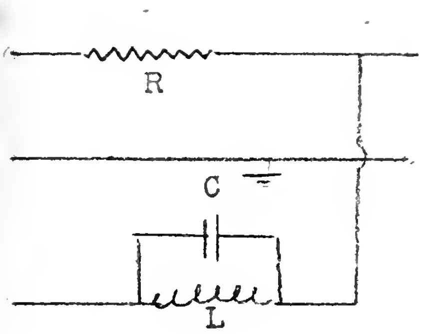
$$\frac{\partial K}{\partial R_k} = -\frac{\mu R_\ell \left\{\left[\frac{r_p + R_\ell}{g_m} + 2 R_k r_p + R_\ell R_k - R_k^2\right] - R_k\left[2 r_p + R_\ell - 2 R_k\right]\right.}{\left[\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot\right]^2} = 0$$

$$\frac{r_p + R_\ell}{g_m} + R_k^2 = 0 \qquad R_k = \pm j \sqrt{\frac{r_p + R_\ell}{g_m}}$$

The expression could also be made to hold for a negative value for rp.

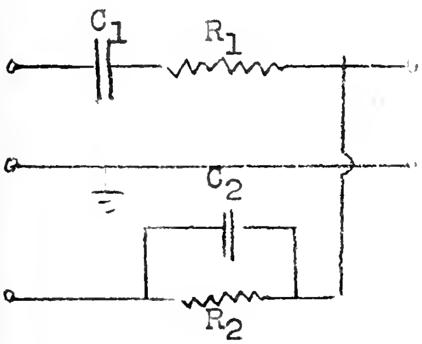
Figure 30

Reduction of Several Phase Shift Networks to Basic Form

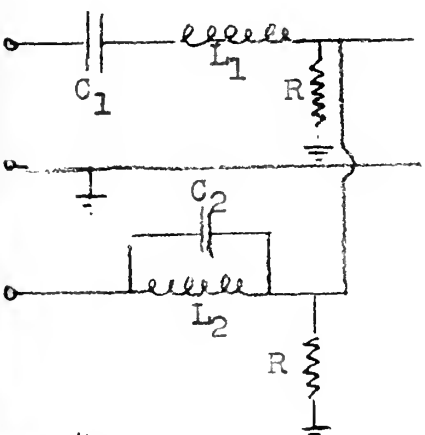


$w^2 = 1/LC$
$Q^2 = CR^2/L$
$k = 1/2$
$L = R/wQ$
$C = Q/wR$

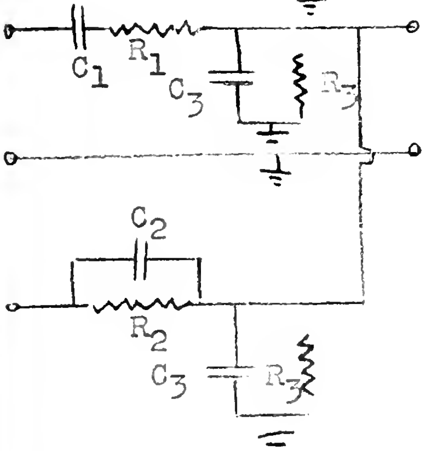
w is resonant frequency and subscript omitted in notation



$w = 1/R_1C_1 = 1/R_2C_2$
$Q = 1/R_2/R_1+2$
$k = 1-2Q$
$R_1 = R_2/(1/Q-2)$
$C_2 = 1/wR$
$C_1 = (1/Q-2)/wR_1$



$w^2 = 1/L_1C_1 = 1/L_2C_2$
$Q = L_1/C_1R^2 = C_2R^2/L_2$
$k = 1/2$
$L_1 = QR/w$
$C_1 = R/wQR$
$L_2 = R/wQ$
$C_2 = Q/wR$



$w = 1/R_1C_1 = 1/R_2C_2 = 1/R_3C_3$
$Q = 1/(R_2/R_1-2) = 1/R_3/R_2+2)$
$k = (1-2Q)/(1+2Q)$
$R_2 = R_3/(1/Q-2)$
$R_1 = R_3/(1/Q-4)$
$C_3 = 1/wR$
$C_2 = (1/Q-2)/wR_2$
$C_1 = (1/Q-4)/wR_3$

APPENDIX V

Precision Capacitance Goniometer

The physical configuration of this device and its equivalent circuit are shown in Figure 28.

Writing the expression for the output current

$$i_{out} = \frac{e}{Z} = \frac{e_1 - e}{1/j\omega C_1} + \frac{e_2 - e}{1/j\omega C_2} + \frac{e_3 - e}{1/j\omega C_3} + \frac{e_4 - e}{1/j\omega C_4}$$

The expression for the brance currents reduce to

$$e/j\omega Z = e_1C_1 + e_2C_2 + e_3C_3 + e_4C_4 - e(C_1 + C_2 + C_3 + C_4)$$

$$e_1C_1 = E_m \sin\omega t(C_0 + C_m\cos\theta)$$

$$= E_mC_0\sin\omega t + \frac{E_mC_m}{2}\left\{\sin(\omega t+\theta) + \cos(\omega t-\theta)\right\}$$

$$e_2C_2 = -E_mC_0\cos\omega t + \frac{E_mC_m}{2}\left\{\sin(\omega t+\theta) - \cos(\omega t-\theta)\right\}$$

$$e_3C_3 = -E_mC_0\sin\omega t + \frac{E_mC_m}{2}\left\{\sin(\omega t+\theta) + \sin(\omega t-\theta)\right\}$$

$$e_4C_4 = E_mC_0\cos\omega t + \frac{E_mC_m}{2}\left\{\sin(\omega t+\theta) - \sin(\omega t-\theta)\right\}$$

$$e/j\omega Z = 2E_mC_m\sin(\omega t+\theta) - 4eC_0$$

Therefore the voltage across the load becomes

$$e = \frac{j\omega Z}{4C_0 + j\omega Z}\left\{2E_mC_m\sin(\omega t+\theta)\right\}$$

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A study of phase measuring circuits and techniques